EXTRAORDINARY TEACHERS

The Essence of Excellent Teaching

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On Becoming a Contructivist Mathematics Teacher

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My journey as a mathematics teacher has spanned the past four decades. It began when I was an undergraduate teaching assistant in astronomy labs at the University of Minnesota. It has included several years as a high school mathematics teacher. In more than thirty years as a mathematics teacher educator at UGA, it has included teaching mathematics in grades K–16 with students ages six to sixty!

Through my teaching, I've experienced many of my most significant professional moments. In this brief essay, I'll try to share the details of certain particular events related to my teaching and address some of the meanings of those experiences. In doing so, I'm trying to portray the transformations in my teaching and in my personal theories upon which I base my teaching.

Moments of Love and Lust

While an undergraduate at the University of Minnesota, I chose to be a school mathematics teacher, because of my love for mathematics. In my early years as a teacher, I was totally dedicated to showing my school mathematics students the power and magic of mathematics. I wanted all of them to understand and appreciate what I had found in mathematics. For example, when I taught geometry, I exerted great energy and enthusiasm for the beautiful proofs and constructions of classical Euclidean geometry. I even experimented with inductive approaches—using a "lab" or measurement approach to discovering all of the theorems about circles, angles, arcs, and chords. I knew that I was effective at explaining and modeling, and, despite my lust, most of my students appeared to be successful. Lust? Indeed, for I focused my efforts on getting my students to know and appreciate the mathematics—but just as I knew its beauty, excitement, fulfillment. I treated mathematics as a finished display, like a classic Greek marble sculpture, rather than contexts for challenging experiences to be creative, generative, involved. I wanted their understanding to be like my understanding.

Early in my career, I encountered two significant events that would affect deeply my perspectives as a mathematics teacher. In my first year of teaching, I participated in a special evening course for seventy-five Twin Cities–area mathematics teachers at Control Data Corporation. We learned to construct FORTRAN programs for simple mathematical procedures—a laborious, error-prone process. Yet, as a novice I caught a glimmer of the potential of the context for promoting something very powerful in mathematics—"teaching the computer to perform a procedure that was *your* design!" When I joined the mathematics faculty at University High School the next year, our mathematics teachers learned of a brand-new computer language called BASIC, and we began using it to stimulate our students to build, test, "debug," and refine or extend their own computer algorithms for most topics of the curriculum. This became a major theme of my research and development activities, which has continued here in my role as a mathematics teacher educator. The infusion of computing tools into school mathematics teaching and learning is now a major goal and pedagogical emphasis in all of our courses—and it continues to be a major problem area of our field.

In that same early period, I participated in the curriculum reform efforts known as "the new math." While this reform is often criticized today, it was a shift toward teaching mathematics for understanding, and it emphasized deep conceptual analyses of fundamental mathematical constructs. Clarity of meaning was to result from being precise in our concepts and language (e.g., the distinction between "number" and "numeral," or the use of unifying ideas such as sets or structural properties). These notions had an impact on my own conceptual understandings of mathematics, and as I taught with these emphases, I experienced the joy of being able to illuminate with greater clarity and precision the details of my subject while also revealing the overarching theory. Further, I was impacted by much greater attention to student activity in the learning process, more "hands on," guided discovery. These were fundamental shifts in the way I was thinking of the psychology of learning and teaching, and these impacts still serve as a major emphasis in my work as a teacher-educator. In helping teachers to become more effective in their teaching, I strive that they will reflect deeply upon the meanings of basic mathematical concepts and then ponder what kinds of problematic situations can be posed to students to stimulate and support their development of such meanings and understandings.

Moments of Inspiration and Excitement

In 1968, I joined the newly created Department of Mathematics Education at The University of Georgia. It was truly an exciting period of growth in our embryonic field, when serious research and scholarship to address fundamental questions of mathematics learning and teaching were established.

The role of the computer, and then the handheld calculator, in school mathematics became even more important questions. First, distributed processing via time-sharing terminals and then the introduction of the microcomputer for classroom-based work fueled the potentials, and my teaching was impacted and changed with each technological hardware and software innovation. While the emphasis upon teaching structure in the "new math" was replaced by curricular designs based on neo-behaviorist hierarchies of performance objectives, in my teaching I continued with a central interest in mathematical problem-solving, discovery, and reasoning processes.

In my teaching of teachers, I enacted these ideas by treating my explorations of computing in terms of the design of mathematical procedures as problematic situations—computer programs were operational embodiments of the student's reasoning about an algorithmic problem. With such programs, students and teachers could then go on to explore and investigate with speed and accuracy well beyond what could happen normally. I found that students and teachers found great excitement in "teaching the computer," and in many cases it inspired the kinds of mathematical questions and inquiries that were not easily provoked with traditional "chalk and talk" teaching.

The results were exciting—such as the time a group of seventh graders came running down the hall, trailing their computer printouts and excitedly announcing that they "had found it." With my help, they had designed a simple program to test any number for primeness, and this had led to getting the computer to help them find "Where do the primes live?" A program was developed to print arrays (sieves) showing asterisks for primes, blanks for composites (and 1). What they had discovered was that, except for 2 and 3, every prime number can be found "in front of, or behind" a multiple of 6. Despite my graduate study of number theory, I did not know this! We went on to find a simple proof of this theorem and eventually to generalize the ideas to sieves of other orders.

When guiding my students to construct a computer program, I found a new context for questioning their thinking and for focusing upon mental processes. With teachers, this became a way of challenging them to be more directly involved with their own students' mathematical thinking. However, I was not fully aware of the more fundamental shifts occurring within my epistemology, and of the impacts upon my pedagogical views.

Moments of Epistemological Shift

Through a collaborative National Science Foundation (NSF)–supported research project in the late seventies, I became involved in investigating the early mathematical development of children. In a

Soviet-style teaching experiment conducted across two years at Whitehead Road Elementary School, I worked with second graders wherein the basic question was explored: Can children invent their own methods for computation? Arising from my views that older students could, indeed, construct their own algorithms (within the computer programming context), I was certain that if we teachers encouraged and supported beginners, they too would generate methods that were based upon their own meanings. Traditionally, elementary teachers model the textbook (adult) procedure for each whole number operation, and typically children struggle to make sense of the "mysteries" of, for example, borrowing to subtract or the complex maneuvers of long division. We know of widespread failures to achieve acceptable learning of arithmetic. My premise was that children would find ways that embodied what made sense to them.

My struggle was to redefine my role as a teacher. How in the world could I provoke and support such invention? Without reporting the rich details of my experiences, I found that these young children *did* invent their own methods, and that there are clear pedagogical approaches that teachers can take to encourage such thinking. Essentially, one needs to help establish fundamental conceptual meanings (e.g., meaning of the operation of division) and action schemes for modeling the task (e.g., using counting strategies on representations with the task). More important, as a teacher one needs to de-center—get out of the way, step aside, and allow the learner the opportunity to engage the challenge, per se. Wait, watch, listen—intervene to help focus, or to clarify, or to provoke analysis, or to reflect.

In this project work, my colleagues included Ernst von Glasersfeld and Les Steffe, and I encountered their early development of radical constructivism. What I came to realize is that by artistic intuition, I had been striving to become a more constructive mathematics teacher. I shifted in my conception of mathematics as a wonderful body of knowledge "out there," finished, real, to a view of mathematics as a human construction that must be made by each individual person. I realized that by exhibiting and modeling "finished" mathematics, I was essentially "robbing" my students of opportunities to make mathematics.

Moments of Dilemma and Frustration

Adoption of a constructivist epistemology brings with it new dilemmas in teaching. If each student's knowledge is idiosyncratic, then how can there be a science to teaching? Are there any generalizations about teaching that can guide what I do? How can I ever plan for a teaching episode if the fundamental need is for me to attend to the personal construction of each student? Indeed, how can I even begin to manage a classroom where everyone is building his or her own knowledge uniquely? How can I, on the one hand, be responsible and accountable for learning but also accept that I can never be so? How can I cover the required content, when a more constructive approach will surely be slower and take longer? Given their prior experiences in learning mathematics, how can I get my students to accept and shift to the personal responsibility implied by constructivism?

What about explaining, or telling—is there no place for this? If the goal is to activate the mental schemes of the child, how can I be sure what I'm doing even "fits"? If Vygotsky's "zone of proximal development" applies, how do I determine this and then teach in ways that reflect lots of different zones among my students? Is there no social dynamic for building up mathematics, or must it be an isolated independent matter? What, then, is the fundamental nature of the teaching-learning relationship? The premises of constructivism can appear to be an impossibility within classroom teaching of mathematics.

Moments of Pedagogical Growth

As a teacher educator, I work with both pre-service and in-service teachers of school mathematics. Teachers in grades K–12 bring their own backgrounds of experience in mathematics and its teaching. Predictably, these experiences seem to be largely traditional—almost never have they had any challenge or opportunity to approach a mathematical situation as their own construction, per se. When I seek to have them experience such constructive approaches, they sometimes resist or rebel. "Why are you expecting us to do this? Just tell us clearly what we have to know for the test, and I'll show you I can do it! Beside, kids can never do mathematics on their own—the teacher must show and tell so they can learn to imitate correctly." Expectations—of oneself and of one's students—can be a major determinant for shaping how one chooses to learn or chooses to teach others. Confronting one's expectations is a major challenge if we are to transform the nature of school mathematical experience.

At the root of this confrontation for me are fundamental questions: Why do we teach school mathematics? (It is the only subject that now spans all grades, preK–12.) What should be the nature of students' experiences? For most of the teachers who study with me, their answers often involve goals of preparing their students for post–highschool education or work. To them, students need to learn basic arithmetic, algebra, and geometry, with perhaps a bit of statistics, probability, and trigonometry in order that they can do well on the SAT or can do basic mathematics on job entry tests. Some teachers express ideas about "getting them to think" or learning to solve applied problems. Almost never do they see mathematical experiences as intrinsic to the formative development of intellective thinking, per se. The notions of Piaget, the eminent theorist of developmental psychology who characterized the development of thought in terms of mathematical structures and processes, don't appear for most classroom teachers as a basis for the goals of mathematical education.

Finding Balances in Mathematics Teaching

Mathematics is among the most dreaded (and sometimes hated) school subjects. Many adults respond with notable negative energy, expressing their fear and dislike for their memories of their own school mathematics experiences. By contrast, mathematics has a widespread reputation as an important subject that can have great value in determining educational and career opportunities and choices. The emotional dimensions of mathematical experiences appear to be significant factors in how people feel about mathematics, and as such, may well be overwhelming determinants of how mathematics students approach their learning of the subject.

In my journey as a mathematics teacher, I've seen myself explore, question, and alter how, why, and even what I teach. Today, I'm deeply convinced that the quality of the experiences of our mathematics students is the fundamental issue and dilemma. Social, economic, and political forces increasingly pressure mathematics teachers to cover more and achieve higher standardized test scores. Despite evidence arising from international comparisons of mathematics achievement and practice, we seem unwilling to acknowledge that "less is more." In Japan and many other countries, mathematics teachers often take much longer to help students develop new ideas; U.S. curricula cover many more topics in less time with more focus on extensive practice at the expense of conceptual understanding—a curriculum that has been said to be "a mile wide, an inch thick." For most mathematics teachers (and myself as a teacher-educator), the struggle to change is a matter of finding balances among conflicting perspectives and demands while the nature of doing mathematics is undergoing radical changes due to the ubiquity of powerful computer tools. In the twenty-first century we will surely see major transformations of school mathematics, its goals, contents, and teaching. I'm eager for my personal journey to continue!

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Larry L. Hatfield (1990 Meigs Award) is Professor and Head, Department of Mathematics Education, College of Education, The University of Georgia. He was raised in the 1940s and 1950s on a Minnesota family farm, and jokingly says that he is "from Lake Wobegon"! He attended a one-room rural schoolhouse through grade seven and strongly believes that his conceptions of teaching were greatly influenced by the one teacher who effectively taught all (typically fewer than a dozen) of the children in grades one through seven. He has taught mathematics and mathematics education at all levels and in many places, including the University of Minnesota, the International Schools of The Hague, Teachers College-Columbia University, Western Australian Institute of Technology, and East Tennessee State University. While serving for two years as Program Director and Deputy Division Director, National Science Foundation, he says that his major challenge was to teach government bureaucrats about quality mathematical education.

As revealed in his essay, Dr. Hatfield's philosophy and epistemology are those of a transforming constructivist. In his role as a teacher-educator, he seeks to help teachers confront their own beliefs about the nature of mathematics, how they see themselves as mathematicians and mathematics teachers, and their preconceptions of their students' thinking. He promotes a problematic approach using powerful computer tools—emphasizing that within the processes of investigating and searching for solutions to nonroutine problems, students will have profound opportunities to understand concepts, discover generalizations, and reflectively appreciate themselves as thinking humans. His theory of mathematics teaching is that artistry as a teacher must be constructed by the individual, and that every great teacher is reflectively searching and re-searching his/her own practices. A primary goal of great teaching is to connect, deeply, with the thinking of the students—and this is also the primary dilemma of teaching.