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Guest Editorial…

In Pursuit of a Focused and Coherent School Mathematics Curriculum

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Most, if not all, readers are familiar with the criticism of a typical U.S. mathematics curriculum being "a mile wide and an inch deep" (Schmidt, McKnight, & Raizen, 1997). A recent analysis by the Center for the Study of Mathematics Curriculum (Reys, Dingman, Sutter, & Teuscher, 2005) reaffirms the crowdedness of most state mathematics standards. However, criticism of U.S. mathematics curricula is nothing new.

In April 2006, the National Council of Teachers of Mathematics (NCTM) released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. This document is an attempt by NCTM to initiate a discussion on what mathematical ideas are important enough to be considered as "focal points" at a particular grade level. But why is it so difficult to have a focused and cohesive school mathematics curriculum? Besides, what makes a curriculum focused and cohesive? In this paper, I would like to offer my opinions on what a focused and cohesive mathematics curriculum may look like and discuss some obstacles for producing such a curriculum.

What makes a curriculum focused?

Clearly, a crowded curriculum naturally tends to be unfocused. A major cause for the crowdedness of many U.S. textbook series seem to be the amount of "reviews," topics that have been discussed at previous grade levels. Some amount of review is probably necessary and helpful. However, in many cases, the topics are redeveloped as if they have not been previously discussed. For example, in teaching linear measurement, most of today's textbooks follow this general sequence of instruction: (a) direct comparison, (b) indirect comparison, (c) measuring with arbitrary or non-standard units, and (d) measuring with standard units. Often, the discussion of linear measurement in Grades K, 1, and 2 textbooks involve all four stages of measurement instruction at each grade level. In contrast, a Japanese elementary mathematics course of study (Takahashi, Watanabe, & Yoshida, 2004) discusses the first 3 stages in Grade 1, and the discussion in Grade 2 focuses on the introduction of standard units. Most Grade 2 textbooks, therefore, start their discussion of linear measurement by establishing the need for (and usefulness of) standard units through problem situations in which the use of arbitrary units is not sufficient.

This redevelopment of the same topic in multiple grade levels may be both the symptom and the cause of a misinterpretation of the idea of a "spiral curriculum." In the past few years, several elementary mathematics teachers who are using a "reform" curriculum told me that it is acceptable for children to not understand some ideas the first time (or even the second or third time) since they will see it again later. Such a view does not describe a spiral. Rather, it seems to be based on the belief that, by introducing a topic early and discussing it often, students will come to understand it. This view is incompatible with a focused curriculum.

However, simply removing some topics from any given grade level does not necessarily result in a focused curriculum. If all items on a given grade level receive equal amount of attention, regardless of mathematical significance, then the curriculum lacks a focus. The *Focal Points* (NCTM, 2006) present three characteristics for a concept or a topic to be considered as a focal point:

- Is it mathematically important, both for further study in mathematics and for use in applications in and outside of school?
- Does it "fit" with what is known about learning mathematics?
- Does it connect logically with the mathematics in earlier and later grade levels? (p. 5)

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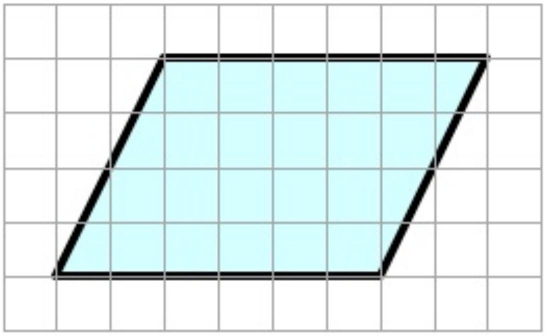

Figure 1. Find the area of this parallelogram (without using the formula).

Whether or not we agree with this particular set of characteristics, if a curriculum is to be focused, it must be based on a set of explicitly stated criteria for organizing its contents.

What makes a curriculum coherent?

It goes without saying that a coherent mathematics curriculum must have its contents sequenced in such a way that a new idea is built on previously developed ideas. Most agree that mathematics learning is like putting together many building blocks. Of course, there is typically more than one way to put together ideas. However, a cohesive curriculum and, ultimately, teachers must have a vision of how learners can build a new idea based on what has previously been discussed. This idea seems to be so obvious, but it is also very easy to overlook.

Furthermore, I believe that textbook writers have the responsibility to make clear the potential learning paths they envision to support teachers who use their materials. This is where many U.S. mathematics textbooks seem to fall short. Too often, teachers' manuals are filled with many suggestions without explicitly discussing how the target ideas may be developed from ideas previously discussed. Thus, teachers are left with an overwhelming amount of information without any guidance regarding how it can be organized and put to work.

Another important factor that contributes to the coherence of a mathematics curriculum is how one part of a curriculum relates to another. For example, the *Focal Points* (NCTM, 2006) states that, in Grade 4, students are to "develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems" (p. 16). However, in the "Connections to the Focal Points", the document also states, "Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts" (p. 16). Therefore, when students are developing fluency with multiplication procedures, the curriculum writers and teachers must pay attention to the products of the assigned problems to insure they will be in the appropriate range. As not all products of two 3-digit numbers will be less than 100,000, these two statements together suggest that the focus of a curriculum should be on helping students understand how and why their multiplication procedures work, rather than focusing solely on students' proficiency with multiplying two 3-digit numbers.

The coherence of a mathematics curriculum is also influenced by its mathematical thoroughness. For example, in many elementary and middle school mathematics curricula, students are asked to find the area of the parallelogram like the one shown in Figure 1. It is expected that most students will cut off a triangular section from one end and move it to the other side to form a rectangle, whose area they can calculate. This idea is discussed in *Principles and Standards for School Mathematics* (NCTM, 2000) as well. Based on this experience, most textbooks will then conclude that the formula for calculating the area of a parallelogram is *base* × *height*. However, this is an overgeneralization. For example, if this is the only experience students have, they will not be able to determine the area of the parallelogram shown in Figure 2, unless they already know the Pythagorean theorem. As a result, students cannot conclude that any side of a parallelogram may be used as the base to calculate its area.

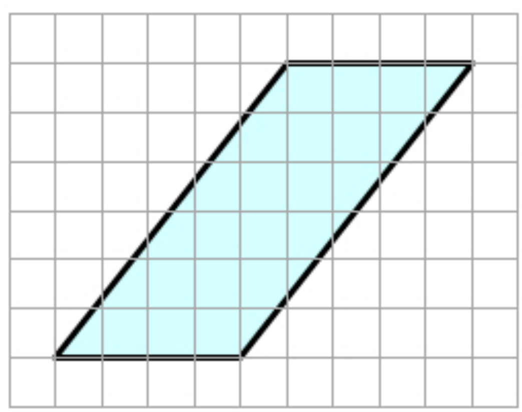

Figure 2. We must change the orientation of this parallelogram to use the method we used previously.

However, we will then need the Pythagorean theorem to determine the lengths of the base and the height.

Therefore, for a curriculum to be cohesive, students should be provided with the opportunity to determine the area of the parallelogram like the one shown in Figure 2. Figure 3 shows some of the ways students may calculate its area. Some of these methods suggest that we could indeed use the horizontal side as the base if we consider the height to be the distance between the parallel lines containing the two horizontal sides.

In addition to having a thorough sequence of mathematical ideas, the coherence of a curriculum may be enhanced by the selection of learning tasks and representations. For example, in a Japanese textbook series (Hironaka & Sugiyama, 2006), the following four problems were used in Grade 6 units on multiplication and division of fractions:

- With 1 *dl* of paint, you can paint $\frac{3}{5}$ m^2 of boards. How many m^2 can you paint with 2 *dl* of paint?
- With 3 *dl* of paint, you can paint $\frac{4}{5}$ m^2 of boards. How many m^2 can you paint with 1 *dl*?
- With 1 *dl* of paint, you can paint $\frac{4}{5}$ m^2 of boards. How many m^2 can you paint with $\frac{2}{3}$ *dl* of paint?
- With $\frac{3}{4}$ *dl* of paint, you can paint $\frac{2}{5}$ m^2 of boards. How many m^2 can you paint with 1 *dl?*

By selecting the same problem context, this particular textbook series hopes that students can identify these problem situations as multiplication or division situations, even though fractions are involved. We know from research (e.g., Bell, Fischbein, & Greer, 1984) that this decision is not trivial for students. Once the operations involved are identified, the series asks students to investigate how the computation can be carried out.

A consistent use of the same or similar items across related mathematical ideas is not limited to the problem contexts. Another way the coherence may be enhanced is through the consistent use of representation. Figure 4 shows how Hironaka and Sugiyama (2006) use similar representations as they discuss multiplicative ideas across grade levels. In early grades, the representations are used primarily to represent the ways quantities are related to each other but, later on, students are expected to use the diagrams as tools to solve problems.

Why has it been so difficult to produce a focused and cohesive curriculum?

We can probably list many different reasons to answer this question. For example, there is a general

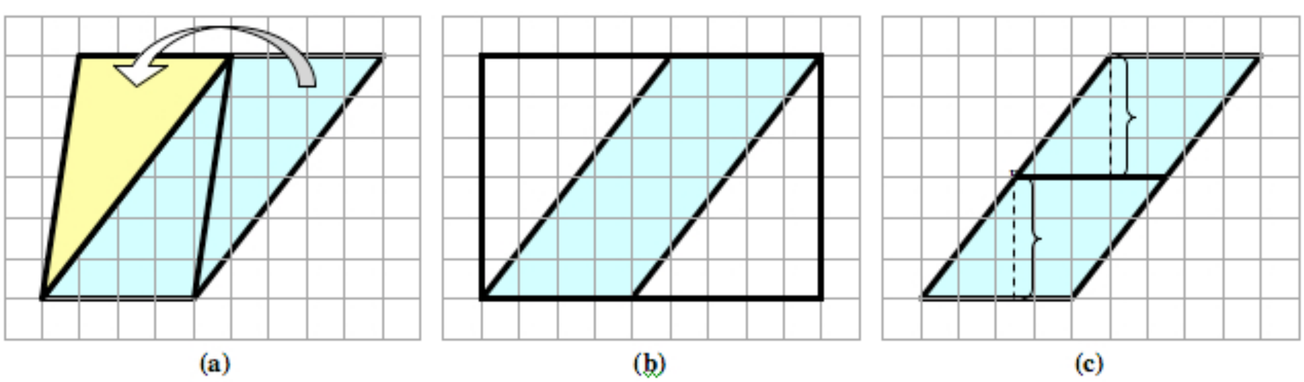

Figure 3. Three methods to determine the area of the parallelogram given in Figure 2. In (a), the given parallelogram is transformed into the familiar parallelogram in which a triangular piece may be cut off and moved to form a rectangle. In (b), the given parallelogram is split into two smaller ones, each of which is a familiar parallelogram. In (c), a large rectangle is drawn to include the parallelogram. By sliding the unnecessary triangular pieces together, we note that the area of the parallelogram is the same as the area of the rectangle on its base (the horizontal side).

reluctance to remove any topic from an existing curriculum. Thus, today's curricula include many ideas that probably were not included 50 years ago, yet virtually all topics from 50 years ago are still included in today's curricula as well. However, I would like to discuss another idea that may be undermining our efforts to create a focused and cohesive curriculum: the lure of replacement units.

The idea of replacement units, high quality materials used in place of a unit in a textbook series, may have started with a good intention. Some reform curriculum materials appear to be created so that parts of the curricula may be used as replacement units. Although many are indeed of very high quality, replacement units may have encouraged the compartmentalization and rearrangement of topics within a curriculum as necessary. Thus, a publisher may be able to "individualize" their textbook series to match different state curriculum standards. If multiplication is introduced in Grade 2 in one state but in Grade 3 in others, there is no problem. One can simply package the introduction of multiplication unit in the appropriate grade level. However, it should be very clear that a focused and cohesive curriculum is much more than simply a sequence of mathematics topics that match the curriculum standards. In addition, as NCTM (2000) states, a curriculum is more than just a collection of problems and tasks (p. 14). One must pay close attention to the internal consistency and coherence of curriculum materials. A Japanese textbook series (Hosokawa, Sugioka, & Nohda, 1998) warned against teachers changing the order of units presented in the series. This is a stark contrast to a rather casual approach that some in this country seem to possess.

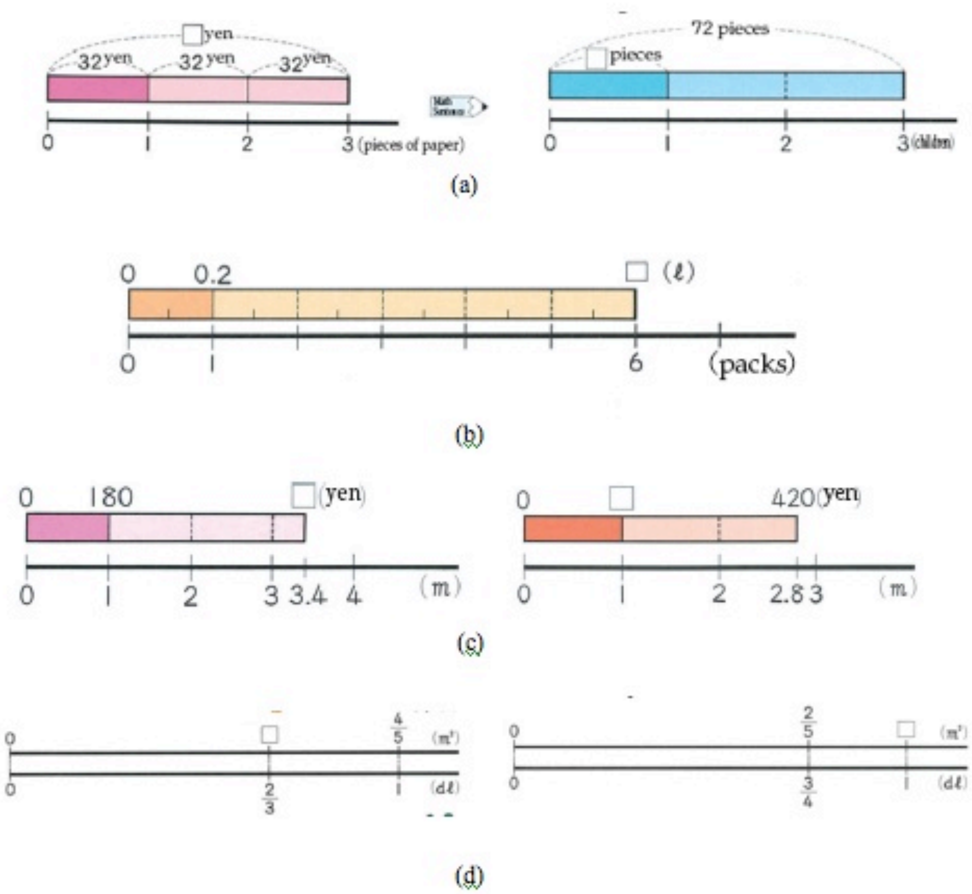

Figure 4. Consistent use of similar representations from Hironaka & Sugiyama (2006): (a) multiplication and division of whole numbers in Grade 3; (b) multiplication of a decimal number by a whole number in Grade 4; (c) multiplying and dividing by a decimal number in Grade 5; and (d) multiplying and dividing by a fraction.

What will it take to produce a focused and coherent curriculum?

The most obvious response to this question is closer collaboration among teachers, researchers, and curricula producers. In Japan, such collaboration is achieved through lesson study. Although lesson study (e.g., Lewis, 2002; Stigler & Hiebert, 1999) is often considered to be a professional development activity, it also serves a very important role in curriculum development, implementation, and revision in Japan. At the beginning of a lesson study cycle, teachers engage in an intensive study of curriculum materials. The participating teachers ask questions such as,

- Why is this topic taught at this particular point in the curriculum?
- What previously learned materials are related to the current topic?
- How are students expected to use what they have learned previously to make sense of the current topic?
- How will the current topic be used in the future topics?
- Is the sequence of topics presented in the textbooks the most optimal one for their students?

During this process, teachers will read, among other things, existing research reports and often invite researchers to participate as consultants. After this intensive investigation of curriculum materials, the group develops a public lesson based on their findings. The public lesson is both their research report and a test of the hypothesis derived from their investigation. Through critical reflection on the observation of public lesson, the group produces their final written report. Japanese textbook publishers often support local lesson study groups, and the reports from those groups are carefully considered in the revision of their textbook series.

Moreover, teachers examine the new curriculum ideas carefully through lesson study. Through this experience, teachers gain a deeper understanding of these new ideas, and they explore effective ways to teach them to their students. Because researchers, university-based mathematics educators, district mathematics supervisors, and even the officials from the Ministry of Education regularly participate in lesson study open houses, lesson study serves as an important feedback mechanism for curriculum development, implementation, and revision.

Lesson study is becoming more and more popular in the United States; however, the involvement by mathematics education researchers and curriculum developers is still rather limited. Moreover, the examination of curriculum materials is often limited as well. A closer collaboration between classroom teachers engaged in lesson study and mathematics education researchers and other university-based mathematics educators is critical if U.S. lesson study is to become a useful feedback mechanism to produce a more focused and coherent school mathematics curriculum.

References

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effect of number size, problem structure and context. Educational Studies in Mathematics, 15, 129-147.

Hironaka, H. & Sugiyama, Y. (2006). *Mathematics for Elementary School*. Tokyo: Tokyo Shoseki. [English translation of *New Mathematics for Elementary School 1*, by Hironaka & Sugiyama, 2000.]

Hosokawa, T., Sugioka, T., & Nohda, N. (1998). *Shintei Sansuu* (Elementary school mathematics). Osaka: Keirinkan. (In Japanese).

Lewis, C. (2002). *Lesson study: Handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.

Reys, B. J., Dingman, S., Sutter, A., & Teuscher, D. (2005). *Development of state-level mathematics curriculum documents: Report of a survey*. Columbia, Mo.: University of Missouri, Center for the Study of Mathematics Curriculum.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas in the world's teachers for improving education in the classroom*. New York: Free Press.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision: An investigation of U.S. science and mathematics education*. Dordrecht, The Netherlands: Kluwer.

Takahashi, A., Watanabe, T., & Yoshida, M. (2004). *Elementary school teaching guide for the Japanese Course of Study: Arithmetic (Grades 1-6)*. Madison, NJ: Global Education