Quadratics and Higher-Degree Polynomials
Finding Roots—Maximum Rectangle Area

Quadratic functions can model relationships other than projectile motion. In this activity you will find an equation relating the area of a rectangle to its width. You will also look at real-world meanings for the \( x \)-intercepts and the vertex of a parabola.

MAKE A CONJECTURE

Suppose you have 24 meters of fencing material and you want to use it to enclose a rectangular space for your vegetable garden.

Q1 What dimensions should you use for your garden to have the largest area possible for your vegetables?

INVESTIGATE

1. Open the TI-Nspire document Maximum Rectangle Area.tns on your handheld and go to page 1.2.

2. You should see a rectangle with a fixed perimeter of 24 centimeters. Drag vertex \( C \) or \( D \) to see different dimensions of the rectangle. Record the lengths and widths on page 1.3. Get at least eight different rectangles. It is okay to have widths that are greater than their corresponding lengths.

3. Calculate the area of each rectangle.

Q2 From the table, what is your guess for the largest area?

4. Go to page 1.4 to see a scatter plot of your \((\text{widths, areas})\) data.

Q3 From the scatter plot view, has your guess changed? If so, what is your new guess?
In the next problem, you will use the same rectangle, but the lengths, widths, and areas will be captured automatically when you drag the vertex.

5. Go to page 2.1 and drag vertex \( C \) or \( D \) around to gather many data points. As you drag, the data will be captured on page 2.2.

6. Go to page 2.3 to see the scatter plot of these \((widths, areas)\) data.

Q4 Write an expression for \( lengths \) in terms of \( widths \). You can determine this algebraically or use the scatter plot of \((wid, len)\) data on page 2.4. To find the slope for your expression using two points on the scatter plot, double-click the \( x \)-coordinate of the given point and enter a new width. The cursor will jump to the nearest point on the scatter plot.

Q5 Using your expression for the length from Q4, write an equation for the area of the garden in terms of the width.

7. Go back to page 2.3, and enter this equation on the scatter plot: choose Text from the Actions menu and click in an empty space to open a text box. Type your equation, using \( x \) and \( y \), and press \( \cdot \). Press \( d \) to put the text tool away, then drag the equation to an axis. Press \( \cdot \) to draw the graph.

Q6 Trace to find the exact largest area: choose Graph Trace from the Trace menu. Place a point on the graph (you may have to press \( \uparrow \) or \( \downarrow \) to trace the graph instead of the scatter plot). Press \( \text{esc} \) to put the trace tool away. Drag the point toward the top of the graph until an M (for maximum) appears. What is the maximum value of area? At what width does this occur?

Q7 Locate the points where the graph crosses the \( x \)-axis by double-clicking the \( y \)-coordinate of the trace point and entering a new one. To get the other \( x \)-intercept, move toward it and repeat the process.

Q8 Explain the meaning of the \( x \)-intercepts in this situation.
Finding Roots—Maximum Rectangle Area

Adapted from Discovering Algebra by Jerald Murdock, Ellen Kamischke, and Eric Kamischke.

Objectives: Students will write equations that model data from a geometric situation.

Activity Time: 25 minutes

Materials: Maximum Rectangle Area.tns

Mathematics Prerequisites: Students should have some number sense involved with measurement and drawing of rectangles; they should be familiar with finding the perimeter and area of a rectangle.

TI-Nspire Prerequisites: Students should be able to open and navigate TI-Nspire documents, graph functions, and enter data. (See the Tip Sheets.)

TI-Nspire Skills: Students will trace functions.

Notes: This activity can be done in multiple ways depending on the skill level of students and amount of time you allocate. Before students open the TI-Nspire document, you might have them fill out a table of lengths, widths, and areas by hand. If students have trouble with this part, they can also draw the rectangles. Use 24-cm lengths of string for kinesthetic learners. A width or length of zero is not acceptable as a measurement, but these are useful values to list. Regardless of how students get the data, have them find and record the length, width, and area of several possible rectangles. Students could also graph the data by hand instead of using the scatter plot on page 1.4 of the TI-Nspire document.

MAKE A CONJECTURE

Q1 There could be a variety of guesses for the maximum area.

INVESTIGATE

2. When students are dragging the vertex of the rectangle to make it change, make sure they don’t stop when the width becomes greater than the length. If they stop too soon, they will only get half of the parabola. Instead of referring to the values as strictly length and width, you might refer to them as two consecutive sides.

3. Students can calculate the areas by hand or by typing the formula into the formula cell for column C.

Q2 The guess will be determined by which values students used as widths. Sample data:

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>35</td>
</tr>
</tbody>
</table>

Q3 This answer might change depending on how close the values are that were chosen from earlier steps.

5. The rectangle’s dimensions of length and width are shown rounded to the nearest whole integer, although the table captures more exact values.

Q4 length = 12 − width. Some students might benefit from solving the equation 2length + 2width = 24 for length. Make sure that the pattern makes sense to students.

Q5 \[ area = width \cdot (12 - width) \], or \[ area = 12width - width^2 \]

Q6 The maximum area is 36 cm² at a width of 6 cm. The 6 by 6 rectangle is actually a square.

Q7 (0, 0) and (12, 0)

Q8 The rectangle has no area if the width is 0 cm or 12 cm.
DISCUSSION QUESTIONS

• How will the equation for areas change if the perimeter is changed?
• How can you find the vertex of the parabola if you know only the x-intercepts?
• What does the vertex of the parabola have to do with the maximum of the function?
• Is the maximum area the “best” answer to the original problem? What assumptions have been made about the situation?

EXTENSION

Have students graph other relationships in the data, such as (widths, perimeters), (lengths, areas), and so on, and find equations for these relationships. (Students will need to create a new list for perimeters.)
Factored Form—Roots and Lines

The factored form of a quadratic equation is \( y = a(x - r_1)(x - r_2) \). The form helps identify the roots of the equation, \( r_1 \) and \( r_2 \). This activity will help you discover connections between a quadratic equation in factored form and its graph.

INVESTIGATE

1. Open a new document on your handheld and add a Graphs & Geometry page.
2. Graph the equations \( f_1(x) = x - 4 \) and \( f_2(x) = x + 3 \).
3. Find the \( x \)-intercepts of each equation: choose Intersection Point(s) from the Points & Lines menu. Place a point at the intersection of each line with the \( x \)-axis. Press \( \text{Esc} \) to put the tool away.

Q1 What is the \( x \)-intercept of each equation?

4. Now graph the equation \( f_3(x) = f_1(x) \cdot f_2(x) \).

Q2 Describe this graph.

Q3 The \( x \)-intercepts of the parabola look very close to the \( x \)-intercepts of the lines. Are they the same? Use tracing to find out: choose Graph Trace from the Trace menu. Press \( \downarrow \) until the trace point is on the parabola. When you are at the intercept, you’ll see a Z on the screen. Are the intercepts the same as those of the lines?

Q4 Make a conjecture about the roots of a quadratic equation and the \( x \)-intercepts of its factors.

5. To test your conjecture, translate and rotate the lines. To translate a line, grab it near the middle. To rotate it, grab it near the end.

Q5 Was your conjecture right? Revise it if not.

Q6 When does the parabola have only one \( x \)-intercept?

Q7 Can you move the lines so that the parabola has no \( x \)-intercepts? Explain.

Q8 Explain anything else you notice about the relationship between the parabola and the lines.
Factored Form—Roots and Lines

Adapted from Discovering Algebra by Jerald Murdock, Ellen Kamischke, and Eric Kamischke.

Objectives: Students will learn that the roots of a quadratic equation can be found from its factored form. They will explore the relationship between the factors of a quadratic equation and its graph.

Activity Time: 20 minutes

Materials: None

Mathematics Prerequisites: Students should be familiar with quadratic equations (vertex form and general form), x-intercepts, and the concept of roots.

TI-Nspire Prerequisites: Students should be able to make a new document and graph equations. (See the Tip Sheets.)

TI-Nspire Skills: Students will construct intersection points, trace graphs, and translate and rotate lines.

Notes: You might do this activity as a whole-class presentation or have students work in pairs.

INVESTIGATE

Q1 \( f_1(x): -3; f_2(x): 4 \)

Q2 The graph is a parabola.

Q3 The intercepts are the same: \( x = -3 \) and \( x = 4 \).

Q4 The roots of a quadratic equation are the same as the x-intercepts of its factors.

5. You might encourage students to translate the lines first, then rotate them. If students move the lines so that the parabola disappears, they can drag the axes inward until the parabola reappears.

Q5 Students may or may not have been right before. Watch for confusion when students are rotating lines. This introduces the multiplier \( a \) of the factored form, but the x-intercepts are still the same. Encourage students to double-check by solving the equations of the lines for the x-intercepts.

Q6 The parabola has only one x-intercept when the lines have the same x-intercept.

Q7 You cannot create a parabola with no x-intercepts using two lines as factors. Explanations might include the fact that the lines each have to cross the x-axis somewhere, unless they are horizontal, in which case multiplying them results in a constant or a linear equation, not a quadratic equation.

Q8 Students may note that moving the lines close together makes a narrow parabola and moving them far apart makes a wide one. They may possibly also notice that the parabola opens down if the lines intersect above the x-axis.

DISCUSSION QUESTIONS

• At some point through the investigation, ask students to name two numbers that have the product of zero. Entertain all ideas, being sure that products of opposites or reciprocals are rejected. Introduce the term zero-product property.

• What can you conclude about \( x \) if you know that \( (x + 3)(x - 4) = 0 \)?
In this activity, you will do an experiment to find a quadratic function to model data. You will collect parabolic data and then find an equation in vertex form that matches the graph.

**EXPERIMENT**

1. Set up the experiment as shown. Prop up one end of the table slightly. Position the motion sensor at the high end of the table and aim it toward the low end.

2. Practice rolling the can up the table directly in front of the sensor. The can should roll up the table, stop about 2 feet from the sensor, and then roll back down.

3. Open a new document on your handheld. Plug the handheld into the sensor. As soon as it is plugged in, you will see a screen similar to the one shown here.

4. Move the cursor to the play button in the small window. When you are ready to collect data, click the play button and gently roll the can up the table. Catch the can as it falls off the table. The sensor should stop collecting data after 5 seconds.

5. The data collected by the sensor will have the form \((\text{time, distance})\). It is collected into two lists named \(\text{run0.time_s}\) and \(\text{run0.dist_m}\). If you did the experiment correctly, you should see a parabolic pattern in the graph. If you need to repeat the experiment, click the play button again and choose OK to rewrite the data.
**INVESTIGATE**

**Q1** Use the table to find the vertex of a parabola that fits your data. If you need to resize the lists to see the data better, tab to the Lists & Spreadsheets application, arrow up to select the column, choose **Resize** from the **Actions** menu, then choose **Column Width**. Press \( \text{ } \) to expand the column, then press \( \text{ } \).

6. Graph your equation: Choose **Text** from the **Actions** menu. Press \( \text{ } \) to open the text box. Type your equation and press \( \text{ } \) again. Then drag the equation to an axis to graph it.

**Q2** How well does your function fit the data? If it doesn’t fit well, try dragging it to adjust it. What is your equation?

**Q3** What is the \( y \)-value if \( x = 7.5 \)? Explain what this point means in words.

**EXPLORE MORE**

1. Expand your equation into general form, \( y = ax^2 + bx + c \), and add it to the graph. Does it match your original equation?

2. Add a Data & Statistics page, make a scatter plot of the data, and perform a quadratic regression on the data. How does this equation compare with the other two? How could you improve the fit of the regression equation?
**Quadratic Motion—Rolling Can**

Adapted from *Discovering Algebra* by Jerald Murdock, Ellen Kamischke, and Eric Kamischke.

**Objectives:** Students will use data collection devices to collect real-world data that can be modeled by quadratic equations. They will then write quadratic equations to model real-world data.

**Activity Time:** 50 minutes

**Materials:** motion sensors, empty coffee cans or large paper rolls, long tables, books; *Optional:* Rolling Can Sample.tns

**Mathematics Prerequisites:** Students should be familiar with quadratic equations including general form and vertex form.

**TI-Nspire Prerequisites:** Students should be able to open and navigate a document. (See the Tip Sheets.)

**TI-Nspire Skills:** Students will use a motion sensor. *Optional:* Students will send documents between handhelds. (See the TI-Nspire Reference Guide.)

**Notes:** You can do this activity in a variety of ways.

**Option 1:** Do the experiment and collect the data as a demonstration, then send the data to students.

**Option 2:** If enough motion sensors are available, have students do the experiment in groups of three or four, then have them share their data within their group.

**Option 3:** Give students the sample data from the Rolling Can Sample.tns document, then proceed with finding the equation to fit the data.

**INVESTIGATE**

**Q1** Sample vertex: (3.2, 0.44). Students might scroll through the data to find the lowest value.

**Q2** Sample equation: \( y = 0.16(x - 3.2)^2 + 0.44. \)

**Q3** For the sample data: 3.398. After 7.5 s, the can is 3.398 m from the motion sensor (and has fallen off the table).

**DISCUSSION QUESTIONS**

- Why is the parabola right side up?
- How is this situation similar to projectile motion? How is it different?

**EXPLORE MORE**

1. The general form of the equation given above is \( y = 0.16x^2 - 0.1024x + 2.0784. \) Graphs of general forms should match students' original graphs.

2. The quadratic regression for the sample data is \( y = 0.12x^2 - 0.79x + 1.80. \) This equation probably doesn't fit the sample data as well as students' other equations. The fit could be improved by deleting the first second of data that doesn't fit the quadratic pattern.

**EXTENSION**

The motion sensor collects velocity and acceleration data, as well as distance and time. You might want to have students explore \((time, velocity)\) and \((time, acceleration)\) graphs and compare them with the \((time, distance)\) graph. The graph of \((distance, velocity)\) is also interesting to explore.
You have developed a great-tasting nutrition drink. You sell it in 12-packs to 20 retail markets in your area. Some of the discount stores resell the 12-packs at a low price in order to sell a large number of packs. Some health clubs sell drinks individually at a high price and sell only a few packs. You have decided to sell your own product at a local festival, but you need to choose a price.

MAKE A CONJECTURE

Q1 Is it better to sell many drinks at a low price or a few drinks at a high price? Explain your ideas.

INVESTIGATE

You decide to test your opinions by collecting data on last month’s sales at each outlet and finding a model to represent the sales and profits.

1. Open the TI-Nspire document Sales and Profits.tns on your handheld and go to page 1.2. You will see data on the selling price per pack from each outlet, the profit they made on each pack, and the total sales for the previous month.

2. Start your research by looking for any patterns in these values. Add two Data & Statistics pages (1.3 and 1.4) and create scatter plots for (sell_price, profit_per) and (sell_price, packs_sold).

3. Find the best line of fit modeling each graph.

Q2 What model did you use for the first graph? What can you learn from this graph’s x- and y-intercepts?

Q3 Give the model for the second graph and explain what the slope in this model tells you.
Q4 Albert’s Market sold 12-packs for $13.50, making a profit of $10.00 on each of the 45 packs they sold. How would you calculate the amount of money Albert’s Market made from this product last month?

4. You want to know which stores made the most money. Go to page 1.2 and add a new variable called endprofit. Give it a formula to calculate this value. (To quickly enter variables in a formula, press \( \text{var} \) and choose Link To.)

Q5 What formula did you use for endprofit? Which outlet made the greatest profit?

Because the best price according to the model may not be one of the prices any outlet charged, you need to look for a formula.

5. Add a Graphs & Geometry page and create a third graph to study how end profit relates to the selling price: Choose Scatter Plot from the Graph Type menu. Press \( \text{tab} \) to choose variables and \( \text{tab} \) to move between them.

6. Choose Function from the Graph Type menu and graph the function \( f(x) = x^2 \). Then choose Window Settings from the Window menu and enter a window that will allow you to see both the parabola and the data.

Q6 Drag the parabola until it fits the data. What model did you find to fit these data? According to that model, what price should you charge at the festival, and what profit will you receive?

Now that you have solved the problem one way, you wonder whether using algebra would give you a solution without dragging a function. You decide to compare the three graphs.

Q7 What are the \( x \)-intercepts of the three models? Explain any patterns you see.

Q8 Expand the equation of your parabola to get the general form, \( y = ax^2 + bx + c \). Then multiply the right sides of your answers to Q2 and Q3. Explain any patterns you see.

Q9 How could you have found the model for sell_price as a function of endprofit without dragging the parabola? What solution would you have gotten?
Objectives: Students will explore how linear models can give information, both graphic and symbolic, about the quadratic model that is their product.

Activity Time: 30–40 minutes

Materials: Sales and Profits.tns

Mathematics Prerequisites: Students should be able to multiply binomials and interpret intercepts on graphs.

TI-Nspire Prerequisites: Students should be able to open and navigate a document, create a scatter plot, use movable lines, define variables using formulas, add function plots to a graph, and trace functions. (See the Tip Sheets.)

TI-Nspire Skills: None

Notes: Step 3 and Q2, Q3, and Q6 give students a chance to find a line (or curve) of fit and interpret the meaning of each function’s terms for this problem situation. Students are gaining experience applying the process of finding a mathematical model to fit a situation, solving the model, then interpreting the result back into the problem situation. Q7 and Q8 give students further experience with looking for patterns by doing calculations.

For a Presentation: Ask several students to interpret the meaning of the constants and the coefficients in the lines of fit. Before you create the graph in step 5, ask students what shape they think the points on the scatter plot will have.

MAKE A CONJECTURE

Q1 Answers will vary widely. You need not reach consensus at this time.

INVESTIGATE

3. Students may add movable lines, use one of the built-in regressions, or find the equation of a line through two representative points. If they write the equation in point-slope form, encourage them to change it to slope-intercept form to facilitate later calculations.

Q2 TI-Nspire’s linear regression gives $y = x - 3.50$, though student values may differ slightly. The $y$-intercept is the per-pack wholesale cost to the retailer. Each item (pack) costs each store $3.50. The $x$-intercept gives sales that would yield a profit of 0. Selling packs at $3.50 would return no profit.

Q3 The regression yields $y = -3.72x + 96.25$, though student values may differ slightly. The slope is the rate at which the number of sales decreases as the price increases. The retailer gets 3.72 fewer sales for each dollar increase in price.

Q4 Multiply $10 per item by 45 items sold to get $450 profit.

Q5 $endprofit = profit\_per\_packs\_sold$; Don’s Beverage #1 and #3 made a profit of $504 for the month.
Q6 The sample fit shown below is approximately \( y = -3.38(x - 15)^2 + 460 \), though student values may differ. The maximum (vertex) indicates that the best price is about $15 per pack, which gives an end profit near $460. That also means that profit per pack is about $15 - $3.50, or about $11.50, and that you will sell about \( \frac{460}{11.50} \approx 40 \) packs total. Exact numbers based on a quadratic regression give a per-pack price of $14.83, selling 40.63 packs and making a profit of $460.33.

Q7 Graph 1: intercept at \( x = 3.5 \) (representing zero profit). Graph 2: intercept at \( x = 25.87 \) (representing zero sales). Graph 3: based on a quadratic regression, intercepts at \( x = 3.42 \) and \( x = 26.23 \) (representing zero profit for either of these reasons). The zeros of the profit function are (approximately) the zeros of its factors. The zeros are not be exactly the same due to the level of estimation involved, but it is important that students understand both the logic behind the relationships of the intercepts and the issues behind problems that show up with the numbers.

Q8 The coefficients of the general form of the quadratic equation should be approximately the same as the coefficients of the product from Q2 and Q3. That is, the profit function is the product of the other two functions.

Q9 The product of the two linear expressions is a quadratic whose zeros are those of the linear functions. The problem could have been solved by graphing the product of the two linear functions to get a price of $14.69 with a profit near $466.
Polynomial Factoring—Maximum Area

Mathematical analysts in business and industry collect data and create models to find maximums, such as the maximum yield or the maximum profit, and minimums, such as the minimum waste or the minimum cost. In this activity, you will solve a similar problem by folding a sheet of paper to find the largest triangle.

EXPERIMENT

1. On a sheet of 8.5 × 11 in. paper, mark each inch from top to bottom along the left 11 in. edge of the sheet.

2. Fold the upper-right corner to one of the marks and crease the paper. There is now a right triangle of a single thickness in the upper-left corner of the page, above the part of the edge that is folded. The two legs of the triangle are along the side and the top of the paper.

Q1 Which mark do you believe will result in the triangle with the largest area?

3. Open a new TI-Nspire document on your handheld. Add a Lists & Spreadsheet page and label the first two columns side and top.

4. Record in the table the lengths of the triangle’s legs as you move the top right corner to marks along the left edge.

Q2 How many marks can you actually use? Explain.

INVESTIGATE

Your goal is to find the exact position for the fold that makes the triangle the largest.

5. Create a new attribute for area, using the formula 0.5 · side · top. To help you see the data, add a Data & Statistics page and create a scatter plot of the (side, area) data.
The graph looks somewhat quadratic. The graph of a quadratic function has
symmetry, with the highest point halfway between the horizontal intercepts.

Q3 Think about how you gathered these data. Where should the horizontal
intercepts be? That is, which values of side would give you 0 area? What point is
halfway between the two side lengths with no area?

Q4 Do you believe these data are actually quadratic? Why or why not?

The easiest type of model to find is linear. Often in statistics, you look for ways to
change the data to “unbend” the curves, then you reverse the process to bend the
line after you have found a model. This sequence is called linearization.

If $z$ is a zero of a function, meaning a horizontal intercept of the graph, then $(x - z)$
is a factor of the function. Because you know two intercepts of this graph, you know
two factors. You can create a data set of lower degree, and therefore one that is more
linear, by dividing the data by one factor.

6. Go to the Lists & Spreadsheet page. Create a new variable called factored and
give it the formula of area divided by one of the factors you know. (Use side
instead of $x$ in your factor.)

7. If the values of factored are linear, then the original data is quadratic. Add
another Data & Statistics page and make a scatter plot of the (side, factored)
data.

Q5 Is this graph linear or curved? Is it increasing, decreasing, or both?

8. Because the data are not yet linear, divide factored by the other factor, creating
factored2. Create a scatter plot of this new variable versus side.

Q6 Choose Add Movable Line from the
Actions menu and adjust the line to
find a linear model for the data points
in this graph.

9. To find the model you’re seeking
for area, work backward. Start with
the equation you found in Q6 and
multiply it by each of the factors you
used to make the data linear. Test this
model by plotting it as a function on
the scatter plot of the (side, area) data.
Polynomial Factoring—Maximum Area
continued

**Q7** What is your model for the area?

10. Add a new Graphs & Geometry page and plot your function from Q7 using $x$ instead of $side$. Hide the entry line by choosing **Hide Entry Line** from the **View** menu.

**Q8** Choose **Graph Trace** from the **Trace** menu. What does tracing the graph tell you about how to fold the paper to get a triangle of maximum area? According to your model, what is that area?

**EXPLORE MORE**

You found models for $factored2$ and for $area$. How can you adjust the model for $factored2$ to get a model for $factored$?
Polynomial Factoring—Maximum Area

Adapted from Exploring Algebra 1 with Fathom by Eric Kamischke, Larry Copes, and Ross Isenegger.

Objectives: Students will use factoring as part of a process of modeling a third-degree polynomial. Students will explore the relationships among intercepts, zeros, and factors as they maximize area in a paper-folding activity.

Activity Time: 20–35 minutes

Materials: 8.5 × 11 in. paper, rulers; Optional: Maximum Area.tns

Mathematics Prerequisites: Students should be able to solve equations and multiply polynomials.

TI-Nspire Prerequisites: Students should be able to create and navigate a document, define variables using formulas in the Lists & Spreadsheet application, make scatter plots, use movable lines, add function plots, and trace. (See the Tip Sheets.)

TI-Nspire Skills: None

Notes: This activity can start with the collection of data using a sheet of paper and a ruler, or you can save time and use the sample data in Maximum Area.tns. You might start by demonstrating how to fold the paper and showing the location of the triangle that students need to measure. If your time is limited and you start with the data in Maximum Area.tns, first demonstrate what is being measured. If students are confused, go back to the physical model, perhaps labeling the side and the top. As you visit working pairs, find one group that divided first by (side − 0) and another that used (side − 8.5). Ask both pairs to be prepared to share.

For a Presentation: If you only have access to one computer with presentation capability, or one handheld and a projection device, you can still ask students to gather the data. Start a table on the TI-Nspire to enter each group's side and top measurements for each inch mark, then use the class average for the presentation. You might plot the value x = 8.5 and talk about Q4 and Q5. Before the student running the computer shows the graphs in steps 7–9, ask what students expect to see. Ask the Explore More question.

EXPERIMENT

Q1 Students will likely pick the 4 in. mark or 4.25 in. (halfway between 0 in. and 8.5 in.). This is a good guess, but it is not as exact as the value they will derive later.

3. If students collect their own data, have them change their document settings to approximate answers: Press , choose System Info, then Document Settings. Tab down to Auto and click to choose Approximate. Then tab to OK and press .

INVESTIGATE

Q2 8; There is no triangle at inch marks below 9 or 10.

Q3 At 0 in. and at 8.5 in. If students have trouble, encourage them to think about a triangle with no area.

Q4 If the graph were symmetric, then the maximum would be at 4.25 in., but it is not. The data are probably not quadratic.

Q5 The graph is not linear. It will be decreasing whether students divide by the factor (side − 0) or by the factor (side − 8.5).

Q6 Answers will vary, depending on the accuracy of students' original measurements. One possible model is factored2 = −0.03side − 0.25.

Q7 Using the sample answer to Q6, area = (side − 0) · (side − 8.5)(−0.03side − 0.25).
Answers will vary. The maximum area occurs when $side$ is about 4.9 in., giving an area of about 7 in$^2$. 

$factored = factored2 \cdot side$, or $factored = factored2 \cdot (side - 8.5)$, depending on the last factor divided out.