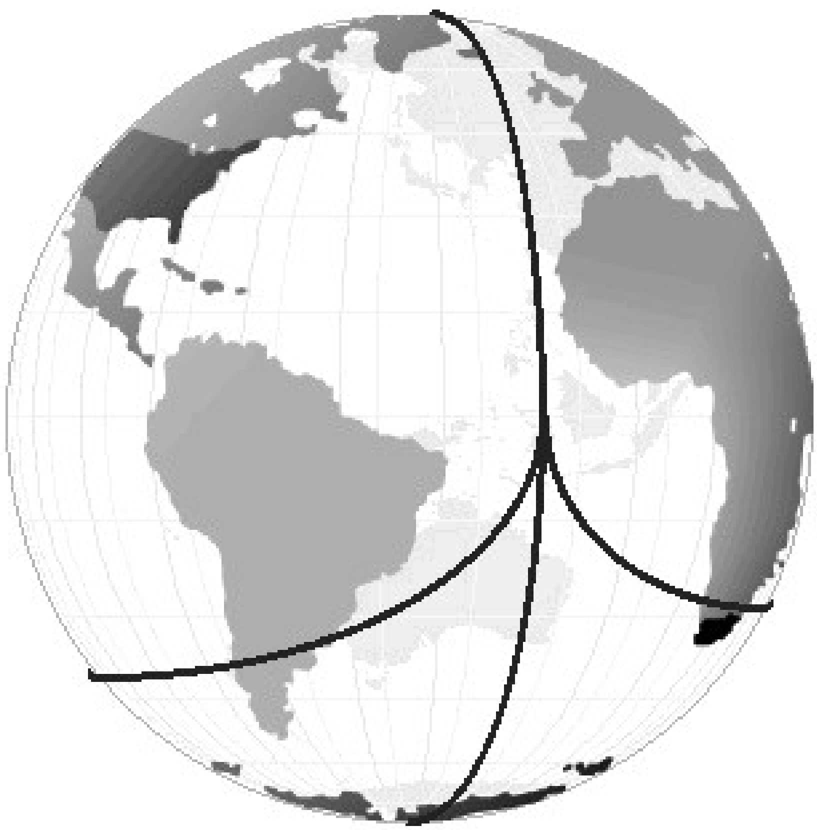
THE MATHEMATICS EDUCATOR

Volume 12 Number 2



FALL 2002

MATHEMATICS EDUCATION STUDENT ASSOCIATION
THE UNIVERSITY OF GEORGIA

A Note from the Editor

Dear readers of *TME*,

It is possible that the publication of the second issue of a volume in October is unprecedented in this journal's history (has *TME* ever come out *ahead* of schedule?). The editorial staff worked extra hard to produce this issue prior to the Psychology of Mathematics Education-North America conference, held in October 2002 at the University of Georgia. An early publication is the first reason this issue is special.

The second reason is that the authors highlight a broad theme of social justice in mathematics education, both within the U.S. and internationally. The trio of pieces from South Africa—Jill Adler's editorial on lessons to be learned from South African contexts, Mamokgethi Setati's argument that language use in multilingual classrooms is necessarily political, and Mellony Graven's discussion of conflicting teacher roles and identities—speak eloquently about the impact of social and political contexts on teachers, learners, and learning in mathematics classrooms. In the second installment of his study, Andy Norton addresses an often-hidden aspect of social context in his call for the need to teach mathematics with respect for and consideration of learners' religious beliefs. Julian Weissglass' provocative questions may push *TME* readers to reexamine their own identities and contexts so that as educators we can teach for equity. Finally, David Stinson provides a book review about the efforts of an educator and activist to work within U.S. contexts in order to give all students an opportunity for an advanced mathematical education. All of these pieces resonate with and reflect the work of mathematics educators around the world who are working for anti-oppressive teaching, learning, and schooling. The editorial staff invites responses from interested readers to any of the pieces in this issue.

As always, I want to thank all reviewers for *TME* because the work in this issue would not be possible without them. The editorial staff is always looking for more reviewers. If you are interested in reviewing for *TME*, please send an email to tme@coe.uga.edu. Please indicate if you have special interests in reviewing articles that address certain topics such as curriculum change, student learning, teacher education, or technology.

I also want to alert all readers to the availability of *TME* online at http://www.ugamesa.org. As you may know, we are now encouraging readers to view *TME* electronically, although hard copy subscriptions for Volume 13 will continue to be $6 for individuals and $10 for institutions. If you currently receive a hard copy and instead would like to be notified by email when a new issue is available online, please send a message to tme@coe.uga.edu. Alternately, if you subscribe online and would like to receive a hard copy, please notify us via email or mail at the postal address below.

Finally, the editorial staff is calling for submissions, particularly from (but not limited to) graduate students. *TME* conducts blind peer review and publishes a variety of manuscripts (see p. 44). We are interested in helping graduate students reach a broader audience with their work and in fostering communication among mathematics educators with a range of professional experience. Please submit!

Wishing you happy and productive reading, learning, teaching, and researching—
Amy Hackenberg

105 Aderhold Hall
The University of Georgia
Athens, GA 30602-7124

About the cover

The globe on the cover was designed by Chris Anderson; the editors are responsible for the "peace yarn" intended to tie the many parts of the world into a whole. To paraphrase Ubiratan D'Ambrosio, mathematics is the only common language of all people; the only common goal of all people is world peace. Therefore, the purpose of mathematics education is to work for world peace.

This publication is supported by the College of Education at The University of Georgia.

THE MATHEMATICS EDUCATOR

An Official Publication of
The Mathematics Education Student Association
The University of Georgia

Fall 2002 **Volume 12 Number 2**

Table of Contents

Upcoming Conferences can be found on page 20.
A subscription form and submissions information are located inside the back cover.

Guest Editorial…

Lessons From and In Curriculum Reform Across Contexts?

Jill Adler

In the past decade, the South African education system has been engaged in the enormous challenge of transformation from a deeply unequal and racially segregated system and curriculum to an integrated system and a new vision for the education and development of all South Africa's learners. The challenge has been at every level of the system, including the structure and functioning of national and provincial departments, districts, schools, and classrooms, as well as the conception and implementation of new policies for curriculum, teacher education, and language-in-education, and for overall funding of education as a public good.

In this short paper, I will look at some of the dimensions of reform in South Africa as they have taken shape in mathematics education. I will point to what I believe are important lessons we are learning, lessons that have wide applicability beyond South Africa's borders. As I have argued in numerous research papers, the South African context brings into sharp relief issues and challenges that are less visible, but equally challenging, in more developed contexts. My discussion is drawn from two research bases: A long-term teacher education research and development project where a group of teacher educators/researchers worked with mathematics, science, and English language teachers to make sense of practice in post apartheid South African education (Adler & Reed, 2002); and a range of critical research that has been undertaken by some doctoral students in the field, including those whose work contributes to this issue (Graven, 2002; Nyabanyaba, 2002; Setati, 2002).

Policy implementation in a context of diversity

National policies are inevitably couched in general terms and thus tend to not engage with the potential and actual consequences of curriculum intentions and their enactment in diverse contexts. Across our projects, we have seen how conceptions of "good" or "best" educational practice can be recontextualized in quite problematic ways. As strategies that work in some school contexts are grafted onto very different contexts and related practices, they can work to undermine the very goals they were intended to address. As we explored teachers' adoption—what we have called take-up—of various practices in support of the new national curriculum, a lesson that emerged for us over and again was that *context matters*. It is a significant challenge for all in education to come to grips with how policies, visions, and goals are themselves a function of how and where they are produced. They will not travel in even ways across different regions and different schooling cultures. Indeed, national policy development needs to find ways of understanding and then promoting diverse practices to meet differing enablements and constraints in diverse conditions.

Language in education policy

I am going to focus on the issue of language in education policy and language practices in multilingual classrooms. While U.S. classrooms might be less obviously multilingual than South African classrooms, linguistic diversity characterizes classrooms everywhere. And linguistic competence is a hidden assumption in the way in which reform in mathematics is being driven. In the U.S., and in related ways in South Africa, mathematics classrooms are to become places where learners engage with rich problems, and they are to do so collaboratively with their peers and their teacher. Teachers are to listen to learners' mathematical thinking and use their verbal and written productions to build on and develop learners' mathematical conceptions.

In bi- and multilingual settings, a specific challenge is immediately visible. How are learners to communicate with each other and the teacher when they do not all share the same main language? In addition, for many learners, their main language is also not the same as the language of instruction and the language in which mathematics texts are produced. We could extend our vision here into all classrooms and ask: How learners are to communicate with each other when there are some for whom clear articulation of

Jill Adler is a Professor in the School of Education and holds the Chair of Mathematics Education at the University of the Witwatersrand, Johannesburg, South Africa, where she teaches a range of graduate courses in Mathematics Education. Jill is currently on sabbatical leave, spending her time in Michigan, first at Michigan State University in East Lansing and then in Ann Arbor at University of Michigan. In both places she is exploring her interest in mathematics professional development, and particularly mathematical knowledge for teaching. Her other research interest is in the field of teaching and learning mathematics in multilingual classrooms.

their thinking is not always easy, and others who might be more reluctant in general to voice their thinking? If we follow this line of thinking, we come to questions like: What are the central purposes for promoting classroom conversation in mathematics? Who is to benefit from this and in what ways? In the fervor that often accompanies visions for change, I believe that we rarely stop to ask: "Is this good for all?" or "Under what conditions, and for what purposes is this 'good'?" Instead, in order to popularize new ideas and ways of working, simplistic panaceas come to frame our discourses.

In South Africa, and in similar multilingual contexts, it is now widely accepted that learners' main languages need to be treated as a resource, rather than a problem. Ways need to be found to enable learners to use their main language as a thinking tool in school. In practical terms, code-switching is advocated as an important pedagogical device. Teachers need to support and enable the switching between learners' main language(s) and the language of instruction. Indeed, this position was explicit across courses in our teacher education program. One of the foci in the related research project was an investigation into teachers' take-up of language practices in support of the new curriculum (Setati, Adler, Reed, & Bapoo, 2002). We worked across primary and secondary schools and across three school subjects—math, science, and English language. We worked with very poor non-urban schools and better-resourced urban schools. And over time we came to see how these level, subject, and regional differences matter, and how differently schools were positioned by what we called their English language infrastructure (Setati, et al.). In non-urban areas, English in and around the school was like a foreign language, used only inside the classroom and rarely heard or spoken anywhere else. We called these "foreign language learning environments", and contrasted them with urban schools where English functioned more like an additional language. In "additional language learning environments", although many learners are not main language English speakers, there is a considerable support in a range of texts in and around the school for the development and use of English. One of the most significant things we learned through the project is just how complex language issues are in non-urban schools. Because there is very limited English infrastructure in the surrounding community on which teachers can build, exposure to English is via the teacher. This puts pressure on teachers to use English as much as possible. Teachers in these schools in the study, particularly across grades 7 to 9, argued quite strongly against frequent code-switching in class. We also found that primary mathematics and science teachers in both urban and non-urban schools feel far more pressure than their secondary colleagues to teach in English because their learners are still in the early stages of learning English.

Further findings from our research also suggested that some of the dominant "messages" in current curriculum documents may need to be reviewed, or at least disaggregated across diverse contexts and settings. For example, one of these messages in *Curriculum 2005* (National Department of Education [NDE], 1997) is that group work is "good" in that it supports exploratory talk and co-operative learning. Code-switching practices facilitate the harnessing of learners' main languages and so facilitate exploratory talk in the classroom. In our research in South Africa, most teachers adopted forms such as group work and so increased the possibilities of *learning from talk* (i.e. of learners' using language as a social thinking tool). Such practice appears to be easily integrated into existing teaching and learning repertoires. However, learning from talk is significantly limited if it is not supported or complemented by strategies for *learning to talk*, i.e. learning subject-specific formal or educated discourses (Barnes, 1992; Mercer, 1995). Across the teachers we worked with we saw unintended consequences of the increasing exploratory talk in class, with teachers either short-cutting or not completing the journey from informal exploratory talk in the main language to formal discourse-specific writing in English. There appears to be a danger that the advocacy of talking to learn and use of main languages is being incorporated or taken up at the expense of learning to talk mathematics or science. In the English language class it may also be at the expense of writing extended texts.

However, in the advocacy for the new curriculum, the issue of how teachers and learners are to navigate the journey from informal exploratory talk (in the learners' main or additional languages) to formal, discourse-specific talk in English, and how they are to do this in contrasting linguistic classroom contexts, is not addressed. This suggests the need for more serious engagement with the possibilities of and constraints on what are typically presented as panaceas for "good practice".

Contexts outside South Africa

There is resonance with this lesson for reform processes and practices elsewhere in research literature that has recently emerged from what are called ESL (English Second Language) contexts in the U.S. In two independent articles reporting research in science and mathematics reform classrooms, Fradd & Lee (1999) and Moschovich (1999) each question whether and how group work and a more facilitative and less instructive role for the teacher actually promote equity goals. In their shared concern for developing discourse-specific talk and competence in learners of mathematics and science, they ask whether so-called universal good practices actually deny rather than enable learning in ESL contexts.

As previously stated, the different English language infrastructures, levels, and subjects in and with which teachers work appear to be significant for shaping Inservice Teacher Education (INSET) possibilities and constraints. We need to disaggregate schools and classrooms along these three different axes and tailor programs according to whether they are within English Foreign Language or English Additional (Second) Language infrastructures, whether they are primary or secondary, whether they are about language as subject or language for a subject. Our concern is that without such specific contextual attentions we will only exacerbate educational inequalities and leave some teachers and learners "stranded" at some point on their educational journey.

Other areas in which context matters

I have devoted most of this editorial to exemplifying a general claim that context matters by looking at language. In our ranging research foci we have found that context matters in other critical areas of mathematics curriculum reform and related teacher education. Briefly, as in many other mathematics reform initiatives, *Curriculum 2005* advocates connections within mathematics and between mathematics and learners' everyday lives (NDE, 1997). Mathematics needs to become more meaningful for learners, and one way of establishing meaning is by embedding mathematical problems in real world contexts. There is also a strong common sense view that this kind of practice will invite more learners into mathematics and thus reduce the inequalities in mathematics performance we currently see when we compare learners from varying socio-economic backgrounds.

However, recent research in the United Kingdom (Cooper & Dunne, 2000) shows how working class children in England experience more difficulty in mathematics assessments that cross the boundary between mathematics and their everyday lives. Many working class learners performed poorly on these kinds of items. When a small number of learners were interviewed, Cooper and Dunne found that working class learners had more difficulty in realizing when an appropriate response could call on their everyday knowledge and experience, and when they had to turn to more explicit mathematical reasoning in answering a problem.

Thabiso Nyabanyaba (2002) followed up on this line of research in the context of school exit examinations in Lesotho. From his involvement in the examining process, Nyabanyaba noticed what seemed like deterioration in performance as more "realistic" items were included in the examination. His research has gone further than Cooper and Dunne to argue that in contexts where success in mathematics significantly determines life chances (like access to jobs and further study), learners select to ignore contextually embedded examination questions. Learners either describe them as "too hard" or as less likely to produce good scores. Learners' reluctance to engage with these items is not because of the difficulty of negotiating the epistemological boundary between mathematical and everyday knowledge. Their choices and responses are more socially determined.

In both of these examples, Cooper and Dunne and Nyabanyaba give us cause to reflect on whether and how connecting mathematics to everyday life is a "universal" means for improved learning and meaning in mathematics in school. They compel us to look and see whether and how questions embedded in real life contexts can become barriers rather than points of access. In a paper just published, Cooper and a colleague (Cooper & Harries, 2002) have indeed looked further: They have explored how different ways of working with embedded problems in a mathematics classroom can produce different enabling conditions. In this paper they argue that, with adjustments to the way such problems are presented, diverse learners might be more willing to negotiate the boundary and so use their everyday knowledge to enhance their experiences of mathematics in school. Again, context matters here. Mathematics curriculum reform, like language in education policy, needs to be thought through in its potential to create both advantage and disadvantage across ranging contextual conditions.

Mathematics teacher education reform

The third and final example I will briefly discuss relates to mathematics teacher education reform. In South Africa we have new *Norms and Standards for Educators* (NDE, 2000) that details multiple roles and competencies required for teaching. These reflect, and gladly so, an acknowledgement of the complexity of the task of teaching. However, we could see in these guidelines the potential for diminished attention to subject-specific (e.g. mathematical) knowledge and its growth for teaching. At the same time, we have been concerned with a strong message coming from national government and its aligned research project (Taylor & Vinjevold, 1999) that subject knowledge alone accounts for teachers' ability to demand high level thinking of their learners. We have argued that, at the moment, there is a pendulum swing in teacher education policy in South Africa between a focus on pedagogical strategies and contextual issues without careful links to how these do or do not support conceptual learning, and a focus on conceptual knowledge that ignores the complexities of transforming this knowledge into appropriate opportunities for learning in school classrooms (Adler, Slonimsky, & Reed, 2002). Mathematics teacher education in South Africa faces critical challenges in reconceptualizing what constitutes mathematical knowledge for teaching at various levels across the school curriculum, and how this might be acquired in teacher education.

Graven (2002, and in this issue) undertook an in-depth study of teacher learning involving 18 teachers over 18 months. Working with the notion of learning as participation in a community of practice both theoretically and practically, Graven shows how teachers with varying mathematical backgrounds all benefited enormously from an INSET program organized to produce and then be supported by a community of practice. Alongside this positive general outcome were also quite specific lessons. Teachers' mathematical histories mattered critically in how they were able to benefit from the mathematical learning opportunities in the program. From this and the teacher education research project referred to in the earlier examples, I conclude again that there can be no panacea, no single kind of project that suits all teachers' needs and areas of development in relation to their subject knowledge for teaching.

National education departments will be concerned to demonstrate improvements in educational performance and so seek out what appear as quick fixes or clear notions of best practice. However, these political desires fly in the face of our growing understanding of, and working with, complex and diverse on-the-ground realities. Indeed, if diverse realities are not carefully attended to, then programs in support of a vision for a new educational order might well undermine their own intentions.

REFERENCES

Adler, J., & Reed, Y. (2002). *Challenges of teacher development: An investigation of take-up in South Africa*. Pretoria: Van Schaik.

Adler, J., Slonimsky, L., & Reed, Y. (2002). Subject-focused inset and teachers' conceptual knowledge-in-practice. In J. Adler & Y. Reed (Eds.), *Challenges of teacher development: An investigation of take-up in South Africa* (pp. 135-152). Van Schaik: Pretoria.

Barnes, D. (1992). The role of talk in learning. In K. Norman, (Ed.), *Thinking voices* (pp. 123-128). London: Hodder & Stoughton.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Cooper, B., & Harries, T. (2002). Children's responses to contrasting 'realistic' mathematics problems: Just how realistic are children ready to be? *Educational Studies in Mathematics*, *49*(1), 1-23.

Fradd, S., & Lee, O. (1999). Teachers' roles in promoting science inquiry with students from diverse language backgrounds. *Educational Researcher, 29*(5), 14-20.

Graven, M. (2002). *Mathematics teachers' learning and communities of practice: The centrality of confidence*. Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners*. Clevedon: Multilingual Matters.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11-19.

National Department of Education. (1997). *Curriculum 2005: Lifelong learning for the 21st century*. Pretoria: NDE.

National Department of Education. (4 February 2000). *Norms and standards for educators* (Government Gazette, Vol. 415, No. 20844). Republic of South Africa: NDE.

Nyabanyaba, T. (2002). *Examining examinations: Trends in the O level mathematics examination in Lesotho and effects on students' epistemological access to mathematics*. Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Setati, M. (2002). *Language practices in multilingual classrooms in South Africa*. Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Setati, M., Adler, J., Reed, Y., & Bapoo, A. (2002). Incomplete journeys: Code-switching and other language practices in multilingual classrooms in South Africa. *Language and Education, 16*, 128-149.

Taylor, N., & Vinjevold, P. (1999). *Getting learning right*. Johannesburg: Joint Education Trust

Researching Mathematics Education and Language in Multilingual South Africa

Mamokgethi Setati

This paper explores policy, practice, and research issues that relate to the teaching and learning of mathematics in multilingual classrooms of South Africa. The paper begins with a brief history of language-in-education policy in South Africa to show how such policy is driven by political as well as educational interests. Thus the paper sets up what will be the main argument: Language-use in a multilingual educational context like South Africa is as much, if not more, a function of politics[1] as it is of communication and thinking. The relationship between language and mathematical learning is considered from a range of perspectives, drawing from a range of literature in the field not confined to South Africa. I will focus specifically on code-switching in multilingual mathematics classrooms, as it is this practice that has been the object of recent research in South Africa. This review of theoretical and empirical work will point to the significance of language as power in mathematics education settings and thus demonstrate the need for researching the relationship between language and the teaching and learning of mathematics in South African classrooms from a political perspective.

The history of language-in-education policy in South Africa

The history of language-in-education policy in South Africa is controversial, particularly regarding the language of learning and teaching (LoLT) in African[2] schools. This history has been interwoven with the politics of domination and separation, resistance and affirmation (African National Congress [ANC], 1994).

The LoLT history in African education can be traced back to the policies of missionary education during the 19th century. In mission schools English featured strongly as a LoLT as well as a school subject. This English as LoLT policy in missionary schools was continued by government-aided African education following the Union of South Africa in 1910 (Beukes, 1992). The importance of learning in the main language gradually came to be recognized in Natal and also in the Cape Province (Hartshorne, 1987). Between 1910 and 1948 the language in education policy was flexible, and different provinces made their own decisions about languages of learning. For instance, in Natal the LoLT in African schools was Zulu for the first six years of schooling (i.e., up to and including Grade 6) until 1948 (Hartshorne).

Language in education during Apartheid

When the Nationalist government came into power in 1948, African schools were removed from provincial administrations and placed under the National Department of Bantu Education. In 1949 the Nationalist government appointed a Commission on National Education, chaired by Dr. Eiselen. At the end of two years, the commission recommended a rigid *mother tongue instruction* policy in the name of Christian National Education (Hartshorne, 1987). The commission recommended that

> … all education should be through the medium of the mother tongue for the first four years, and that this principle should be progressively extended year by year to all eight years of primary schooling (p. 68).

However, the government did not follow the Eiselen report closely, largely because of its concern with protecting and expanding the influence of the Afrikaans language in the system (Hartshorne).

In 1953 the government passed the Bantu Education Act which stipulated that mother tongue instruction be phased in across all primary school Grades in African primary schools, with English and Afrikaans as compulsory subjects from the first year of schooling. At the time, English and Afrikaans were the only two official languages. The latter language had developed out of Dutch settlement. In addition, both English and Afrikaans were also to be used as languages of learning and teaching on a 50/50 basis when transfer from main language[3] learning took place in the first year of secondary school (Hartshorne, 1987). The educational interests of the pupils became subordinate to ideological and political factors. The government's greatest concern at the time was that the constitution of South Africa required equality in treatment of the two official languages. These policies were centered on fears that the Afrikaner language, culture, and tradition

Mamokgethi Setati recently completed her Ph.D. at the University of the Witwatersrand, Johannesburg, South Africa, and is now a senior lecturer in mathematics education at the same university. Her main research interest is in discursive practices in mathematics teaching and learning in multilingual settings. She is the national president of the Association for Mathematics Education of South Africa (AMESA).

might be overwhelmed by the older, more internationally established English language, culture and tradition (Reagan & Ntshoe, 1992).

Alongside these policies for African learners, white, so-called "coloured", and Indian schools were also segregated along apartheid racial lines but came under different legislation. Learners in these schools were required to take both English and Afrikaans throughout the 12 years of school, one at a first language level, and the other at either first or second language level. Depending on department and location, the LoLT in these schools was either English or Afrikaans, or in some cases dual medium. As English and Afrikaans were the main languages of white, coloured, and Indian learners, these learners were able to learn through the medium of their main language in both primary and secondary schools.

Hartshorne (1987) has argued that the language policy in African education in South Africa since the 1948 election (and particularly since the Bantu Education Act) has centered on two major issues: mother tongue instruction and the establishment of the primacy of Afrikaans as the preferred LoLT in secondary school. The majority of the African people rejected both these issues. Though not unmindful or ashamed of African traditions per se, mainstream African nationalists have generally viewed cultural assimilation as a means by which Africans could be released from a subordinate position in a common, unified society (Reagan & Ntshoe, 1992). Therefore, they fought against the use of African languages in schools, since their use was seen as a device to ensure that Africans remain "hewers of wood and drawers of water" (p. 249).

The LoLT issue became a dominating factor in opposing the system of Bantu Education during the apartheid era. African opinion never became reconciled to the extension of first language learning beyond Grade 4 nor the dual medium policy (of English and Afrikaans) in secondary school (Hartshorne, 1987). Many analysts trace the 1976 uprising, which began in Soweto and spread all over the country, to rather belated attempts by the Nationalist government to enforce the controversial and highly contested 50/50 language policy for African learners that was first promulgated in 1953. This policy prescribed that all African children at secondary school should learn 50% of their subjects in Afrikaans and the other 50% in English. African teachers were given five years to become competent in Afrikaans.

In 1979, in the wake of the 1976 revolt, the government introduced a new language policy. This new policy emphasized initial main language learning with an eventual shift in the LoLT to English or Afrikaans. As a general rule, the African child began his or her schooling in the main language, which remained the LoLT through the fourth year of schooling (Grade 4). During these first four years both English and Afrikaans were studied as subjects. Beginning in the fifth year of schooling (Grade 5), there was a shift in the LoLT to either English or Afrikaans, the official languages of the country.

In 1990 the Nationalist government passed an amendment to the 1979 Act giving parents the right to choose whether their child would be immediately exposed to a second language (e.g., English) as the LoLT (from Grade 1), or would experience a more gradual transfer. While there is no systematic research evidence, it is widely held that many schools with an African student body adopted English as the LoLT from Grade 1 (Taylor & Vinjevold, 1999).

The unbanning of liberation movements and the release of Nelson Mandela in February of 1990 signalled the beginning of a new era for South Africa. The ANC was voted into power in 1994 and multiple policy initiatives began across all social services. In terms of language policy, a process to fully recognize the rich multilingual nature of South Africa was initiated. The constitution adopted in 1996 for a post-apartheid South Africa recognizes 11 official languages. For the first time nine African languages—Sesotho, Sepedi, Setswana, Tshivenda, Xitsonga, IsiNdebele, IsiXhosa, IsiSwati and IsiZulu—received official status, in addition to English and Afrikaans. In 1997 a new language-in-education policy that recognizes 11 official languages was introduced.

Language in education in the new South Africa

According to this policy, not only can South African schools and learners now choose their LoLT, but there is a policy environment supportive of the use of languages other than one favored LoLT in school, and so too of language practices like code-switching. While this new language-in-education policy is widely acknowledged as "good", it is already meeting significant on-the-ground constraints. Recent research suggests that most schools are not opting to use learners' main languages as LoLTs in both policy and practice (Taylor & Vinjevold, 1999). This situation is not unexpected; as described earlier, main language as LoLT policy or mother tongue instruction has a bad image among speakers of African languages. It is associated with apartheid and hence inferior education.

While the new language policy in South Africa is intended to address the overvaluing of English and Afrikaans and the undervaluing of African languages, in practice English continues to dominate. Even though English is a main language of a minority, it is both the language of power and the language of educational and socio-economic advancement, thus it is a dominant symbolic resource in the linguistic market (Bourdieu, 1991) in South Africa. The linguistic market is embodied by and enacted in the many key situations (e.g., educational settings, job situations) in which symbolic resources, like certain types of linguistic skills, are demanded of social actors if they want to gain access to valuable social, educational, and eventually material resources (Bourdieu).

Various institutional arrangements and government policies continue to produce the dominance of English in the linguistic market. First, the LoLT in higher education institutions is either English or Afrikaans, and it seems that this policy will continue for many more years since it has not yet been challenged in higher education circles. Second, there is an English/Afrikaans-language pre-requisite for anyone aspiring to become a professional in South Africa. Students need to pass a school-leaving examination in English as a first or second language, in addition to mathematics, to enter and succeed in the English-medium training programs in professional fields such as medicine and engineering and in order to earn qualifications to enter high-income professions. "The symbolic market is therefore not a metaphor but one with transactions that have material, socio-economic consequences for individuals" (Lin, 1996, p. 53).

Third, there are still policies upholding English as an official, legal, and government language. The nine African languages spoken by the majority of South Africans are still secondary to English in reality; for example, most of the policy documents are written in English only. Fourth, there is imposition of an English-language requirement for individuals aspiring to join the civil service. For instance, ability to communicate in English is one of the requirements for anyone willing to train for police or military service. The fact remains that English is the most important criterion for selection for high-ranking officials; knowledge of an African language is seen as an additional asset but not an essential one.

With these institutions and policies well-entrenched in the various administrative, educational, and professional arenas of South Africa, a symbolic market has been formed where English constitutes the dominant, if not exclusive, symbolic resource. It is a prerequisite for individuals aspiring to gain a share of the socio-economic, material resources enjoyed by an elite group.

Recognizing the historically diminished use and status of the nine African languages of the people of South Africa, in December 1995 the Minister of Arts, Culture, Science and Technology announced the establishment of a Language Plan Task Group (LANGTAG). Its role was to identify South Africa's language-related needs and priorities. Since then, LANGTAG has articulated a multilingual policy for South Africa. It proposed a widespread use of the nine African languages in all spheres. This proposal was challenged by some members of the division of Applied English Language Studies at the University of the Witwatersrand, who believe that the widespread use of the nine African languages will not necessarily alter the status and power of English (Granville, et al., 1998). They proposed that all learners be guaranteed access to the language of power (English), while at the same time ensuring redress for African languages. They maintain that this redress will enable teachers to teach English as a subject without guilt and to help learners understand that all languages are valuable and are a national treasure (Granville, et al.). The issue of the dominance of English in South Africa is not easy to resolve. As Sachs, a constitutional court judge, pointed out, in South Africa "all language rights are rights against English" (1994, p. 1).

The above discussion highlights the link between language and politics in South Africa. It is clear that in South Africa, change in language-in-education policy has been linked to change in political power. Thus if "mathematics education begins in language, [it] advances and stumbles because of language" (Durkin, 1991), then the politics of changing language policies must impact on mathematical teaching and learning practices particularly in multilingual classrooms. Just like the language-in-education policy, changes in the school curriculum in South Africa have been preceded by changes in political power.

The school mathematics curriculum context of South Africa

In 1995 the Minister of Education announced the introduction of the new curriculum. This curriculum was intended "to overturn the legacy of apartheid and catapult South Africa into the 21st century" (Chisholm, et al., 2000, p. 8). It would bring together education and training, content and skills, values and knowledge. In March 1997 this curriculum was launched and became known as *Curriculum 2005* (National Department of Education [NDE], 1997).

According to *Curriculum 2005* a minimum of two languages should be offered; however, there is no prescription as to what these languages should be. Multilingualism is recognized as a valuable resource. According to the official document,

> The advancement of multilingualism as a major resource affords learners the opportunity to develop and value: their home languages, cultures and literacies; other languages, cultures and literacies in

> our multilingual country and in international contexts; and a shared understanding of a common South Africa (Department of Education [DoE], 1997).

A focus on an integrated and non-disciplinary division of knowledge in *Curriculum 2005* led to an introduction of eight learning areas that replaced school subjects. The understanding here was that learning areas would promote strong integration of what is learned both academically and in everyday life (Chisholm, et al., 2000). The official description of the mathematics learning area is that

> Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction (DoE, 1997).

The above description emphasizes the role that language plays in the expression, development, and contestation of mathematics. This view highlights language as a tool for communication, thinking and politics in mathematics. The role of language in mathematics is also highlighted in the specific outcomes for mathematics. Outcome 9 states that learners should be able to “use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.” In the elaboration of this outcome, the policy documents states that

> Mathematics is a language that uses notations, symbols, terminology, conventions, models and expressions to process and communicate information. The branch of mathematics where this language is mostly used is algebra and learners should be developed in the use of this language.

Curriculum 2005 was reviewed during the year 2000. As a result of the review, a task team was appointed to develop a national curriculum statement for mathematics. Language and communication of mathematics are again emphasized in the national curriculum statement. Learning outcome 2 that focuses on patterns, functions and algebra states, “the learner should be able to recognise, describe and represent patterns and relationships, and solves problems using algebraic language and skills” (Chisholm, et al., 2000).

As the above discussion shows, there is an explicit focus on multilingualism and the communication of mathematics in the present mathematics school curriculum. This focus raises questions about the language used for communication and how mathematics teachers find a balance between making language choices in their multilingual classrooms, advancing multilingualism, and initiating learners into ways of communicating mathematics.

In the remainder of the paper I explore the complex relationship between language and mathematics, drawing on research in South Africa and elsewhere. As stated above, I develop an argument for the centrality of the political for both research and practice in language and mathematics education. Without such a focus we will fail to understand and so work with the demands that teachers face.

The relationship between language and mathematics

In his seminal work, Pimm (1987) explored some of the connections between language and mathematics He argues that one way of describing the relationship between mathematics and language is in terms of the linguistic notion of register.

> The mathematics register is a set of meanings that belong to the language of mathematics (the mathematical use of natural language) and that a language must express if it is used for mathematical purposes….We should not think of a mathematical register as constituting solely of terminology, or of the development of a register as simply a process of adding new words (p. 76).

Part of learning mathematics is acquiring control over the mathematics register—learning to speak, read, and write like a mathematician. The mathematics register includes words; phrases; symbols; abbreviations; and ways of speaking, reading, writing and arguing that are specific to mathematics. Since mathematics is not a language like French or Xhosa, speaking or writing it requires the use of an ordinary language, the language in which mathematics is taught and learned. As discussed earlier, a majority of learners in South Africa learn mathematics in a language that is not their main language. Thus communicating mathematically in multilingual classrooms in South Africa means managing the interaction between the following:

- ordinary English and mathematical English.
- formal and informal mathematics language.
- procedural and conceptual discourses.
- learners’ main language and the LoLT.

The interaction between ordinary English (OE) and mathematical English (ME)

As Pimm (1987) argues, speaking like a mathematician does not just involve the use of technical terms, but also phrases and characteristic modes of arguing that are consistent with the mathematics register. Mathematical speech and writing have a variety of language types that learners need to understand in order to participate appropriately in any mathematical conversation. These types are ordinary and mathematical English, or logical language and meta-language (Pimm; Rowland, 1995). Mathematical English can be described as the English mathematics register, in the same way that we can have mathematical French, or mathematical Swahili. One of the difficulties of learning to use mathematical English is that in its spoken (sometimes also in its written) form it is blended with ordinary English (natural language), and the distinction between the two languages is often blurred. Mathematical English is embedded in the language of predicate logic, which includes items such as "and", "or", "if…then", "some", "any", and so on (Rowland). These words from the language of predicate logic can be confusing when used in mathematical conversations (spoken or written) because they can appear to belong to ordinary English when in fact they have been redefined for logical reasons. Pimm uses the following example to highlight one of the difficulties with the word "any". Consider the following two questions:

a) Is there any even number which is prime?
a) Is any even number prime?

According to Pimm (1987), question a) is clear and the response to it is "yes, 2 is an even number and it is also prime". Question b), however, is not clear and can be interpreted in two conflicting ways:

- Is any (i.e., one specific) even number prime? Answer: Yes, 2 is an even number and it is also prime.
- Is any (i.e., every) even number prime? Answer: No, almost all are not prime.

The source of the difficulty in the above example is the mathematical meaning of the word "any". While the word "any" is used widely in mathematics at all levels, it is ambiguous. It may be used to mean *every* or *some*. For example the question "is any rectangle a rhombus?" can legitimately be answered both "yes, a square is" and "no, unless it happens to be a square". According to Pimm (1987), mathematicians tend to use "any" to mean "every", and on occasion, their meaning conflicts with ordinary usage. However, it is clear from the above examples that the word "any" is not used consistently in mathematical English. The same can be said of other logical connectors such as "if…then". Mathematics words can also mean different things depending on whether they are used informally or in a formal mathematical conversation.

Formal and informal mathematics language

In most mathematics classrooms both formal and informal language is used, in either written or spoken form. Informal language is the kind that learners use in their everyday life to express their mathematical understanding. For example, in their everyday life, learners may refer to a "half" as any fraction of a whole and hence can talk about dividing a whole into "three halves". Formal mathematical language refers to the standard use of terminology that is usually developed within formal settings like schools. Considering the above example of a "half", in formal mathematics language it is inappropriate to talk about a whole being divided into three halves. If any whole is divided into three equal parts, the result is"thirds".

The valued goal in school mathematics classrooms is formal written mathematical language (Setati & Adler, 2001). Pimm (1991) suggests two possible routes to facilitate movement from informal spoken language to formal written mathematical language. The first is to encourage learners to write down their informal utterances and then work on making the written language more self-sufficient. The second is to work on the formality and self-sufficiency of the spoken language prior to writing it down.

I have previously argued that in multilingual mathematics classrooms where learners learn mathematics in an additional language, the movement from informal spoken language to formal written language is complicated by the fact that the learners' informal spoken language is typically not the LoLT (Setati & Adler, 2001; Setati, 2002). Figure 1 shows that the movement from informal spoken to formal written mathematics in multilingual classrooms occurs at three levels: from spoken to written language, from main language to English, and from informal to formal mathematical language. The different possible routes are represented in Figure 1 by different lines. For instance, one route could be to encourage learners to write down their informal utterances in the main language, then write them in informal mathematical English, and finally work on making the written mathematical English more formal. In this case the teacher works first on learners writing their informal mathematical thinking in both languages, and thereafter on formalizing and translating the written mathematics into the LoLT. Another possibility is to work first on translating the informal spoken mathematical language into spoken English and then on formalizing and writing the mathematics. Of course there are other possible routes that can be followed.

As can be seen in Figure 1, while formal written mathematics in the learners' main language(s) is a possibility, there are no routes to or from it. There are a variety of reasons why most mathematics teachers in multilingual classrooms in South Africa would not work on formalising spoken and written mathematics in the main language:

- The mathematics register is not well developed in most of the African languages.
- Due to the dominance of English this work would generally be seen or interpreted as a waste of time.

Procedural and Conceptual Discourses

In addition to both spoken and written modes of formal and informal mathematics, mathematics in school is carried out by distinctive mathematics discourses. For example, Cobb (Sfard, Nesher, Streefland, Cobb, & Mason, 1998) has distinguished calculational from conceptual discourses in the mathematics classroom. He defines calculational discourse as discussions in which the primary topic of conversation is any type of calculational process, and conceptual discourse as discussions in which the reasons for calculating in particular ways also become explicit topics of conversations (Sfard, et al.). Previously I have referred to procedural and conceptual discourses where procedural discourse focuses on the procedural steps to be taken to solve the problem. I have argued for the use of the term procedural discourse rather than Cobb's calculational discourse because "procedural" is self-explanatory (Setati, 2002). To give an example, in the problem 28 + 18, learners can enter into discussions focusing on the procedure (or calculational processes) to follow without focusing on why the procedure works (e.g., why they do not write 16 under the units). Another possibility is that learners can solve this problem by engaging in discussions about the problem and also about why a particular procedure works (conceptual discourse).

In conceptual discourse, the learners articulate, share, discuss, reflect upon, and refine their understanding of the mathematics that is the focus of the interaction or discussion. It is the responsibility of the teacher to arrange classroom situations in which these kinds of interactions are possible—classroom situations where conceptual discourse is not just encouraged but is also valued. The teacher, as a "discourse guide" (Mercer, 1995), conveniently acts to a considerable extent as an intermediary and mediator between the learners and mathematics, in part determining the patterns of communication in the classroom, but also serving as a role model of a "native speaker" of mathematics (Pimm, 1987). As a consequence, from their interactions with the teacher, students learn the range of accepted ways in which mathematics is to be communicated and discussed. The teacher models the accepted ways of acting-interacting-thinking-valuing-speaking-reading-writing mathematically.

Teachers can encourage conceptual discourse by allowing learners to speak informally about mathematics—exploring, explaining, and arguing their interpretations and ideas. The challenge here is for the teacher to know when and how to lead learners from their informal talk to formal spoken mathematics. If the teacher intervenes prematurely, she could unintentionally discourage learners from expressing and exploring their conceptions regarding the mathematics that is being discussed. This kind of exploratory talk is important for learners to develop

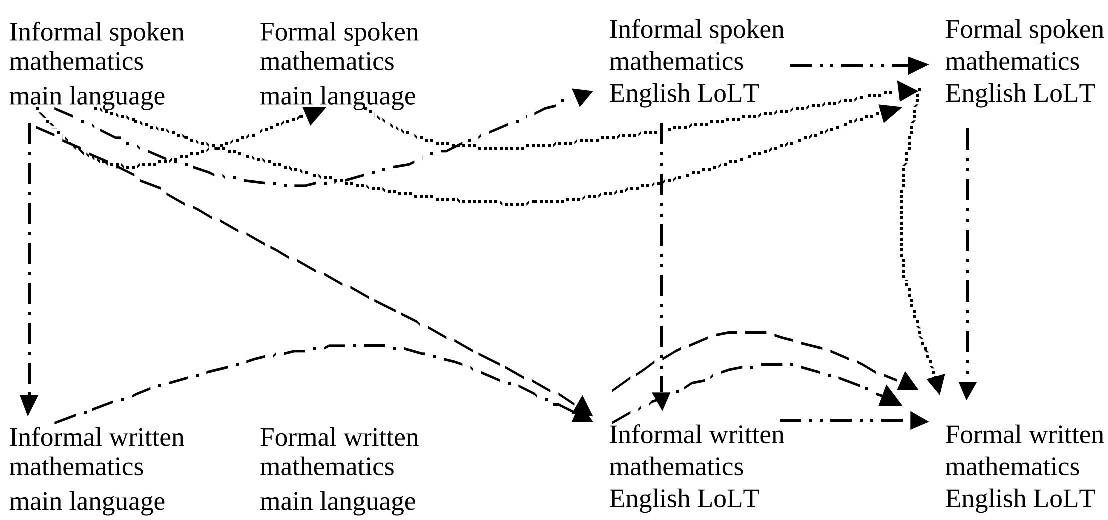


Figure 1. Alternative routes from informal spoken (in main language) to formal written (in English) mathematics language.

ideas and concepts in a comfortable environment. It is also important for enabling teachers to listen to learners' ideas and conceptions so that these can be worked with and built upon (Setati, Adler, Reed, & Bapoo, 2002). It is in this environment of informal exploratory talk that learners begin to acquire conceptual discourse. Therefore the teacher is faced with the challenge of keeping a balance between informal and formal spoken language and of making sure that the learners explore their ideas sufficiently in informal ways in order to acquire fluency in formal conceptual discourse. Adler refers to this challenge as the *dilemma of mediation*:

> The dilemma of mediation involves the tension between validating diverse learner meanings and at the same time intervening so as to work with the learners to develop their mathematical communicative competence (Adler, 2001, p. 3).

This dilemma of mediation highlights a key challenge in the context of *Curriculum 2005*, where learner participation is valued and teachers strive for inclusion, voice, and greater mathematical access. This challenge is exacerbated by the "*dilemma of transparency* where the tension is between implicit and explicit teaching of the mathematics language" (Adler, 2001, p. 4, italics added). As Adler has noted, these dilemmas are a challenge for all teachers. They are not specific to a multilingual classroom. But as this paper will show, these dilemmas are more complex in a multilingual classroom where informal spoken mathematics is not in the LoLT. In these classrooms learners are acquiring English while learning mathematics.

Adler's description of the dilemmas is crucial and highlights the fundamental pedagogic tensions that cannot be resolved once and for all. However, she does not explain in specific detail why teachers experience these dilemmas in the way that they do. This focus was not her project. She posits an explanation that the dilemmas are at once personal and contextual. For instance, one of the teachers in Adler's study experienced the dilemma of mediation because of changes in her classroom and because of her personal commitment to her learners. In this paper I argue that the dilemmas that the multilingual mathematics teachers experience are also political.

The complex and competing demands on mathematics teachers in multilingual classrooms in South Africa are evident from the above discussion. Teachers have to ensure learners' access to English, to the language of mathematics, and to a range of mathematical discourses. In particular, they need to assist learners in developing formal spoken and written mathematics. These competing demands can affect classroom practices in contradictory ways, as evidenced in Adler's identification of two teacher dilemmas.

In the remainder of this paper I explore the implications of policy and the growing understanding of the complex role of language in mathematical learning as I examine research on the teaching and learning of mathematics in bi- and multilingual classrooms.

Teaching and learning mathematics in bi/multilingual classrooms

The effects of bi/multilingualism on learners have been the focus of research for decades. I will not rehearse the arguments here as they have been described in detail elsewhere (e.g., Saunders, 1988; Setati 2002). Instead, the discussion below focuses on the complex relationship between bi/multilingualism and mathematics learning as well as on code-switching as a common learning and teaching resource in many bi/multilingual classrooms in South Africa and elsewhere.

Bi/multilingualism and mathematics learning

The complex relationship between bilingualism and mathematics learning has long been recognized. Dawe (1983), Zepp (1989), Clarkson (1991), and Stephens, Waywood, Clarke, and Izard (1993) have all argued that bilingualism per se does not impede mathematics learning. Their research used Cummin's (1981) theory of the relationship between language and cognition. Cummins distinguished different levels and kinds of bilingualism. He also showed a relationship between learning and level of proficiency in both languages on the one hand and the additive or subtractive model of bilingual education used in school on the other. Secada (1992) has provided an extensive overview of research on bilingual education and mathematics achievement. He pointed to findings of a significant relationship between the development of language and achievement in mathematics. In particular, oral proficiency in English in the absence of mother tongue instruction was negatively related to achievement in mathematics. Rakgokong (1994) has argued that using English only as a LoLT in multilingual primary mathematics classrooms in South Africa where English is not the main language of the learners has a negative effect on the learners' meaning making and problem solving. His study showed that, in classrooms where English was the only language used for teaching and learning, learners were able to engage in neither procedural nor conceptual discourse. Varughese and Glencross (1996) found that students at the university level had difficulty in understanding mathematical terms such as integer, perimeter, and multiple. Their study involved first-year mathematics students in a South African university who were learning mathematics in English, which was not their main language.

This field of research has been criticized because of its cognitive orientation and its inevitable deficit model of the bilingual learner (Baker, 1993). The argument is that school performance (and by implication, mathematics achievement) is determined by a complex set of inter-related factors. Poor performance of bilingual learners thus cannot be attributed to the learners' language proficiencies in isolation from the wider social, cultural, and political factors that infuse schooling.

While I agree with the above criticism, I read into this cognitively-oriented research an implicit argument in support of the maintenance of learners' main language(s), and of the potential benefits of learners using their main language(s) as a resource in their mathematics learning. As Secada (1991) has argued, bilingualism is becoming the norm rather than the exception in urban classrooms. Hence the need in mathematics education research to examine classroom practices where the bilingual speaker is not only treated as the norm, but where his or her facility across languages is viewed as a resource rather than a problem (Baker, 1993). In an article entitled "The Bilingual as a Competent Specific Speaker-hearer", Grosjean (1985) argues for a bilingual (or holistic) view of bilingualism in any consideration of bilinguals. This view is different from the monolingual view, which always compares the linguistic ability of bilinguals with that of monolinguals in the languages concerned. Bilinguals have a unique and specific language configuration and therefore they should not be considered as the sum of two complete or incomplete monolinguals:

> The coexistence and constant interaction of the two languages in the bilingual has produced a different but complete language system. An analogy comes from the domain of athletics. The high hurdler blends two types of competencies: that of high jumping and that of sprinting. When compared individually with the sprinter or the high jumper, the hurdler meets neither level of competence, and yet when taken as a whole, the hurdler is an athlete in his or her own right. No expert in track and field would ever compare a high hurdler to a sprinter or to a high jumper, even though the former blends certain characteristics of the latter two. In many ways the bilingual is like the high hurdler (p. 471).

In Grosjean's terms, language practices in multilingual classrooms will not be the same as in any other classroom. For example, an important aspect of multilingualism, one which makes the multilingual person an integrated whole, is code-switching. As indicated earlier, code-switching is now encouraged by the language-in-education policy. In the section below I present a review of research on code-switching in bilingual and multilingual classrooms in South Africa and elsewhere.

Code-Switching in bilingual and multilingual mathematics classrooms

Code-switching occurs when an individual alternates between two or more languages. Code-switches can be deliberate, purposeful, and political. There are important social and political aspects of switching between languages, as there are between switching between discourses, registers, and dialects. Historically, code-switching in South Africa has had an inferior status (Setati, 1998). As a result, many people still regard it as a grammarless mixture of languages. Some monolinguals see it as an insult to their own rule-governed language. It is generally believed that people who code-switch know neither language well enough to converse in either one alone. Grosjean (1982) points out that it is because of these attitudes that some bi/multilinguals prefer not to code-switch, while others restrict their switching to situations in which they will not be stigmatized for doing so. For instance, in a multilingual classroom learners may choose to switch only when interacting with other learners and not with the teacher.

Why code-switch? Even though code-switching has received substantial criticism from purists, there are researchers who see it as a valuable communication resource. On the basis of their ethnographic observation of classroom interaction in three primary schools in Kenya, Merrit, Cleghorn, Abagi, & Bunyi (1992) argue that code-switching provides an additional resource for meeting classroom needs. Poplack cited in Grosjean (1982) argues that code-switching is a verbal skill requiring a large degree of competence in more than one language, rather than a defect arising from insufficient knowledge of one or the other. Some researchers see code-switching as an important means of conveying both linguistic and social information. For instance, Gumperz cited in Grosjean (1982) maintains that code-switching is a verbal strategy, used in the same way that a skilful writer might switch styles in a short story. For instance, a teacher can use learners' main language as a code for encouragement. By using learners' main language in this manner, the teacher may implicitly be saying to learners "I am helping you; I am on your side".

In most classrooms code-switching seems to be motivated by cognitive and classroom management factors (Adendorff, 1993; Merritt, et al., 1992): Usually it helps to focus or regain the learners' attention, or to clarify, enhance, or reinforce lesson material. Determinants of code-switching in the mathematics classroom are only partially dictated by formal language policy. Even if official policy exists, teachers make individual moment-to-moment decisions about language choice that are mostly determined by the need to communicate effectively:

> Multilingual teachers do not only teach lessons and inculcate values having to do with conservation of resources. They, perhaps unconsciously, are socialising pupils into the prevailing accepted patterns of multilingualism (Merritt, et al., p. 118).

As pointed out earlier, the language-in-education policy in South Africa recognizes eleven official languages and is supportive of code-switching as a resource for learning and teaching in multilingual classrooms. Within this policy environment that encourages switching, it is important that research focus not only on whether code-switching is used or not in the teaching and learning of mathematics but also on how and why it is used or not used.

According to Baker (1993), code-switching can be used to describe changes which are relatively deliberate and have a purpose. For example, code-switching can be used:

- to emphasize a point,
- because a word is not yet known in both languages,
- for ease and efficiency of expression,
- for repetition to clarify,
- to express group identity and status or to be accepted by a group,
- to quote someone,
- to interject in a conversation, or
- to exclude someone from an episode of conversation.

Thus code-switching has more than just linguistic properties; it can also be used for political purposes.

Researching code-switching in multilingual classrooms. Research on code-switching in multilingual classrooms in South Africa reveals that it is used for a variety of reasons. A study undertaken in primary mathematics and science classrooms in the Eastern Cape, South Africa, has shown that code-switching is used to enable both learner-learner and learner-teacher interactions (Ncedo, Peires, & Morar, 2002). Adendorff (1993), who observed non-mathematics lessons in the Kwazulu-Natal province of South Africa, found that an English teacher switched to Zulu in order to advance his explanation of the meaning of a poem. The same teacher also used code-switching as a language of provocation—he used it to raise controversial issues. Most bi/multilingual persons switch when they cannot find an appropriate word or expression or when the language being used does not have the necessary vocabulary item or appropriate translation (Grosjean, 1982). This kind of switching would occur in a bi/multilingual mathematics conversation. For instance, if learners can hold a mathematical conversation in Setswana, it is possible that the mathematical terms will be in English, because mathematics has a well-developed register in English but not in Setswana. While some of the technical mathematics terms are available in Setswana, they are not widely known and used. For instance while the Setswana word for an equilateral triangle is "khutlo-tharo-tsepa", this term is usually not used in mathematical conversations in Setswana. There are instances where the multilingual mathematics learner knows a mathematics word in both English and her main language (e.g., Setswana), but the English word becomes more available during mathematical conversations. This phenomenon can be understood because, as indicated earlier, a majority of African language speakers in South Africa learn mathematics in English.

Code-switching as a learning and teaching resource in bi/multilingual mathematics classrooms has been the focus of research in the recent past (e.g., Addendorff, 1993; Adler, 1996, 1998, 2001; Arthur, 1994; Khisty, 1995; Merritt, et al., 1992; Moschkovich, 1996, 1999; Ncedo, Peires, & Morar, 2002; Setati, 1996, 1998; Setati & Adler, 2001). These studies have presented the learners' main languages as resources for learning mathematics. They have argued for the use of the learners' main languages in teaching and learning mathematics as a support needed while learners continue to develop proficiency in the LoLT while learning mathematics. All of these studies have been framed by a conception of mediated learning, where language is seen as a tool for thinking and communicating. In other words, language is understood as a social thinking tool (Mercer, 1995). Therefore it is not surprising that problems arise when learners' main languages are not drawn on for teaching and learning. Arthur (1994) conducted her study in Botswana primary schools where the main language of the learners is Setswana. English as the LoLT starts from standard six. Her study of the use of English in standard six mathematics classrooms revealed that the absence of learners' main language (Setswana) diminished the opportunities for exploratory talk, and thus for meaning-making. The form and purposes of the teaching and learning interaction in these classrooms were constrained by the use of English only. As Arthur explains, communication was restricted to what she referred to as "final draft" utterances in English, which were seemingly devoid of meaning.

This dominance of English is not unique to Botswana. As discussed earlier, English as the LoLT continues to dominate in multilingual classrooms in South Africa despite the new progressive language-in-education policy (Taylor & Vinjevold, 1999). In describing the code-switching practices of primary school mathematics teachers in South Africa, Setati and Adler (2001) observed the dominance of English in non-urban primary schools. They argued that in these schools English is only heard, spoken, read, and written in the formal school context, thus teachers regard it as their task to model and encourage English. Setati, Adler, Reed, and Bapoo (2002) described these school contexts as foreign language learning environments (FLLEs). They distinguish FLLEs from additional language learning environments (ALLEs), where there are opportunities for learners to acquire the English language informally outside the classroom. The English language infrastructure of ALLEs is more supportive of English as the LoLT. There is more environmental print (e.g., advertising billboards) in English, and teachers and learners have greater access to English newspapers, magazines and television, and to speakers of English. Setati, et al. (2002) found greater use of code-switching in ALLEs.

Code-switching has been observed as a "main linguistic feature in classrooms where the teacher and the learners share a common language, but ha[ve] to use an additional language for learning…the learners' language is used as a form of scaffolding" (National Centre for Curriculum and Research Development, 2000, p. 68). Adler (1996, 1998, 2001) identified code-switching as one of the dilemmas of teaching and learning mathematics in multilingual classrooms. Adler observed that in classrooms where the main language of the teacher and learners is different from the LoLT, there are ongoing dilemmas for the teacher as to whether or not she should switch between the LoLT and the learners' main language, particularly in the public domain. Another issue is whether or not she should encourage learners to use their main language(s) in group discussions or whole-class discussion. These dilemmas are a result of the learners' need to access the LoLT, as critical assessment will occur in this main language. Adler's study suggests that the dilemmas of code-switching in multilingual mathematics classrooms cannot necessarily be resolved. They do, however, have to be managed.

Moschkovich (1996, 1999) argues that bilingual learners bring into the mathematics classroom different ways of talking about mathematical objects and different points of view on mathematical situations. She emphasizes that a discourse approach can also help to shift the focus of mathematics instruction for additional language learners from language development to mathematical content. In Mercer's (1995) terms, the teacher in Moschkovich's study was a discourse guide. As Figure 1 shows, the role of the teacher as a discourse guide in a multilingual mathematics classroom involves moving learners from a stage where they can talk informally about mathematics in their main language(s) to a stage where they can use the formal language of mathematics in the LoLT (English), and can engage in procedural and conceptual mathematics discourses in English.

The above discussion demonstrates that there is a growing amount of theoretical and empirical work related to mathematics teaching and learning in bi/multilingual classrooms. The unit of study in early research on bilingualism was the bilingual learner. It is my view that this location of the problem in the learner was based on an underlying assumption of inferiority—that there is something wrong with the bilingual or multilingual learner. Recent studies have moved from focusing on the bi/multilingual learner to the bi/multilingual classroom. This change in focus drew attention to the significance of the teacher as a discourse guide in the bi/multilingual classroom, and to code-switching and the dilemmas that emerge with its use. All of the studies referred to have been framed by a conception of mediated learning, where language is seen as a tool for thinking and communication.

A different perspective on language. Language is much more than a tool for communication and thinking; it is always political (Gee, 1999). Decisions about which language to use, how, and for what purpose(s), are political. This political role of language is not dealt with in the literature on bi/multilingualism and the teaching and learning of mathematics. My own experience as a multilingual teacher and researcher in multilingual mathematics classrooms suggests that we cannot describe and explain language practices in a coherent and comprehensive way if we stop at the cognitive and the pedagogic aspects. We have to go beyond these aspects and explore the political aspects of language use in multilingual mathematics classrooms. Research so far does not capture this complexity. As mentioned earlier, Adler (2001) points to the complexity by describing dilemmas as personal and contextual, and more particularly by exploring the dilemma of code-switching. According to Adler, teachers in multilingual classrooms face a continual dilemma of whether to switch or not to switch languages in their day-to-day teaching:

> If they stick to English, students often don't understand. Yet if they "resort" to Setswana (i.e., they switch between English and Setswana) they

> must be "careful", as students will be denied access to English and being able to "improve" (p. 3).

Adler (2001) describes the language practices of a teacher in her study (Thandi) as follows:

> Thandi's actions, including reformulation and repetition, were not tied simply to her pedagogical beliefs, but also to her social and historical context and her positioning within it, including her own confidence of working mathematically in English. In particular, in the South African context, where English is dominant and powerful, Thandi's decision-making and practices were constrained by the politics of access to mathematical English. Thandi might value using languages other than English in her mathematics classes to assist meaning-making. But this pedagogical understanding interacts with strong political goals for her learners, for their access, through mathematics and English, to further education and the workplace. In addition, her decision-making on code-switching inter-related in complex ways with the mathematics register on the one hand and its insertion in school mathematical discourses on the other (p. 85).

In my view, Adler partially explains Thandi's dilemma. Thandi experienced the *dilemma of code-switching* not only because of her learners and because of the pedagogical and political contexts but also because of who she is: an African mathematics teacher who shares a main language with her additional language learners. In addition Thandi saw her role not only as a mathematics teacher but also as someone who is supposed to make sure that her learners are prepared for higher education in English and the outside world. Thandi's language practices were tied up with her pedagogy, identity, and understanding of the power of English. Thandi's dilemma of code-switching is thus not only pedagogic but also political. The political and the pedagogic are in tension. This dilemma manifests itself in the multiple identities that teachers take on. For instance, politically Thandi wanted her learners to have access to English, and therefore she did not use code-switching; however, pedagogically she knew that she needed to switch so that her learners could understand and participate in the lesson.

It is clear from the above discussion that there are a growing number of studies that have focused on language use in bi/multilingual classrooms. But none of the studies focused on language as a political tool. How is language used "to enact activities, perspectives and identities" (Gee, 1999, p. ?) in bi/multilingual mathematics classrooms? The main argument of this paper is that research on the use of language(s) in multilingual mathematics classrooms needs to embrace language-in-use as a political phenomenon.

The political role of language in the teaching and learning of mathematics

In South Africa, mathematics knowledge and the English language are social goods. They are perceived to be a source of power and status. Both of them provide access to higher education and jobs. The fact that English is a language of power and socio-economic advancement in South Africa makes English a valued linguistic resource in multilingual mathematics classrooms. Even though the nine African languages now enjoy an official status, they still do not enjoy the same kind of status as English.

Gee (1999) argues that when people speak or write they create a "political" perspective; they use language to project themselves as certain kinds of people engaged in certain kinds of activity. Words are thus never just words; language is not just a vehicle to express ideas (a cultural or communicative tool), but also a political tool that we use to enact (i.e., to be recognized as) a particular "who" (identity) engaged in a particular "what" (situated activity). Thus a mathematics teacher who is also a cultural activist will have an identity that shifts and takes different shapes as she enacts her multiple identities in and through language. Her decisions about what language to use, how, when, and why will be informed by the activity and identity she wants to enact. The point here is that mathematics teachers, like all people, have multiple identities. Research that considers the use of language in multilingual mathematics classrooms only as a pedagogic and cognitive tool does not attend sufficiently to the multiple identities of multilingual teachers.

Fairclough (1995) refers to institutional and social identities. He argues that institutions impose upon people ways of talking and seeing as a condition for qualifying them to act as subjects. That is, institutions impose certain identities on people. For example, to be a mathematics teacher one is expected to master the discursive (ways of talking) and ideological (ways of "seeing") norms which the teaching profession attaches to that subject position. That is, one must learn to talk like a mathematics teacher and see things (i.e., things like learning and teaching) like a mathematics teacher. These ways of talking and seeing are inseparably intertwined in the sense that in the process of acquiring the ways of talking which are associated with a subject position, one necessarily also acquires its ways of seeing (ideological norms). Any social practice can thus be regarded as a speech and ideological community. Mathematics teaching is a speech and ideological community. To be part of this social practice you need to talk and see things like a mathematics teacher. Any social practice imparts ways of talking and seeing that are relevant for that practice. People need this kind of shared knowledge in order to participate in that social practice. In the case of mathematics teaching, a mathematics teacher needs this kind of knowledge in order to say acceptable things in an appropriate way.

Since this shared knowledge is rooted in the practices of socio-culturally defined groups of people, Holland and Quinn as cited in D'Andrade and Strauss (1992) refer to it as culture. When talking about culture in this way, they do not refer to people's customs, artifacts, and oral traditions, but to what people must know in order to act as they do, make the things they make, and interpret their experience in the distinctive ways they do. Thus, they would argue that to be a mathematics teacher, one needs more than mathematics content knowledge—one also needs the cultural knowledge of mathematics teaching. According to Holland and Quinn, this cultural knowledge is organized into schemas that are called *cultural models.* Cultural models are taken-for-granted models of the world that guide people's actions and their expression of values and viewpoints. Gee (1999) argues that cultural models are like tapes of experiences we have had, seen, read about, or imagined. People store these tapes either consciously or unconsciously and treat some of them as if they depict prototypical (what we take to be "normal") people, objects, and events. Cultural models do not reside in people's heads. They are available in people's practices and in the culture in which they live—through the media, written materials and through interaction with others in society.

In a recent study focusing on language use in multilingual mathematics classrooms in South Africa, I have considered language practices in multilingual mathematics classrooms from a political perspective, thus attending to the multiple identities of multilingual teachers. In the study I used the notion of cultural models as an analytic tool to explore and explain the language practices of six teachers in multilingual mathematics classrooms (Setati, 2002). Since cultural models are not only inferred from what people say, but also from how they act, think, value, and interact with others (in Gee's terms, their "Discourses"), these teachers were interviewed and observed in practice.

Three categories of cultural models emerged from the analysis of the interviews and lesson transcripts in that study. *Hegemony of English* cultural models reflect the dominance of English in the teaching and learning of mathematics in multilingual classrooms. The *Policy* cultural models revealed the teachers' understanding of the language-in-education policy. The *Pedagogic* cultural models mirrored the tensions that accompany teaching mathematics to learners whose main language is not the LoLT. These multiple cultural models reveal the multiple identities that teachers enact in their multilingual classrooms to make both mathematics and English, and mathematics in English, accessible to learners. Through these three categories of cultural models, the pedagogical and the political were deeply intertwined.

English is International emerged as the "master model" (Gee, 1999). The emergence of this master model was is not surprising. The dominance of English in politics, commerce, and the media in South Africa is well known. English is seen as a key to academic and economic success, and therefore being fluent in it opens doors that are closed to vernacular speakers (Friedman, 1997). The *Hegemony of English* cultural models that emerged in this study form part of the various institutional arrangements and government policies which, as discussed earlier, have achieved the formation of an English-dominated linguistic market.

In an in-depth analysis of one of the lessons observed, English emerged as a legitimate language of communication during teaching, and thus was the language of mathematics, of learning and teaching and of assessment. However, this dominance of English produced a dominance of procedural discourse, mainly because the learners were not fluent in conceptual discourse in English. Thus whenever the teacher asked a conceptual question, they responded in procedural discourse in English, or remained silent until she changed the question into a procedural one. This dynamic is mainly due to the differing linguistic and mathematical demands of procedural discourse and conceptual discourse. In conceptual discourse learners are not only expected to know the procedure that needs to be followed to solve a problem, but also why, when, and how that procedure works. Procedural discourse, on the other hand, focuses on the procedural steps that should be followed in the solution of a problem. These steps can be memorized without understanding. Unlike conceptual discourse, procedural discourse does not require justification. It is therefore not surprising that in an additional language learning environment like the multilingual classrooms in the study, procedural discourse would dominate when mathematical conversation was in English. As illustrated earlier in Figure 1, the journey from informal spoken mathematics (in the main language) to fluency in formal spoken and written procedural and conceptual mathematics discourses in English is complex in multilingual classrooms.

What is more interesting is that the teacher whose lesson was analyzed was convinced that she was promoting multilingualism in her teaching. The analysis shows that she used the learners' main language for regulation and solidarity. While she was regulating the learners' behavior, she also showed her support and unity with them. Her utterances in the learners' main language were encouraging and motivating to the learners. Her regulatory utterances in English, on the other hand, were more authoritative, giving instructions to and reprimanding learners. Thus the learners' main language was a voice of solidarity while English was the voice of authority.

This study has moved the dominance of English from a common-sense position to a rigorous and theoretical understanding of this dominance, and of how it plays itself out in the multilingual mathematics classroom in terms of creating mathematical opportunities for learners. This study has also revealed how the power of mathematics and English can work together in multilingual mathematics classrooms to reduce the mathematical opportunities for procedural discourse. Further, it appears that for substantial teaching and learning and engagement in conceptual discourse to occur, the learners' main languages are required. However, given the master model of *English is International*, it is not always possible to fulfill this requirement. The issue is not only that additional language learners learn mathematics in a language that is not their main one, but that the various languages used will privilege different discourses of mathematics.

Conclusion

The theoretical elaboration in this article has shown that to describe and explain language practices in multilingual mathematics classrooms, we need to go beyond the pedagogic and cognitive aspects. All language practices occur in contexts where language is a carrier of symbolic power. This aspect shapes the selection and use of language(s) and mathematical discourses. The different ways in which teachers and learners use and produce language is a function of the political structure and the multilingual settings in which they find themselves. A teacher's use of code-switching in a multilingual mathematics class is therefore not simply cognitive or pedagogic, but is also a social product arising from that particular political context.

REFERENCES

Adendorff, R. (1993). Code-switching amongst Zulu-speaking teachers and their pupils. *Language and Education, 7*(3), 141-162.

Adler, J. (1996). *Secondary teachers' knowledge of the dynamics of teaching and learning mathematics in multilingual classrooms.* Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Adler, J. (1998). A Language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics, 18*, 24-33.

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht: Kluwer Academic Publishers.

African National Congress, (1994). *A policy framework for education and training.* Johannesburg: Education Department, The African National Congress.

Arthur, J. (1994). English in Botswana primary classrooms: Functions and constraints. In C. M. Rubagumya (Ed.), *Teaching and researching language in African classrooms* (pp. 63-78). Clevedon: Multilingual Matters Ltd.

Baker, C. (1993). *Foundations of bilingual education and bilingualism*. Clevedon: Multilingual Matters Ltd.

Beukes, A. (1992). The politics of language in formal education: The position of Afrikaans. In I. A. Stevenson (Ed.), *Sociolinguistics for applied linguistics*. Pretoria: University of Pretoria.

Bourdieu, P. (1991). *Language and symbolic power.* Cambridge, MA: Harvard University Press.

Chisholm, L., Volmink, J., Ndhlovu, T., Potenza, E., Mahomed, H., Muller, J., Lubisi, C., Vinjevold, P., Ngozi, L., Malan, B., & Mphahlele, L. (2000, May 31). *A South African curriculum for the twenty-first century.* (Report of the review committee on Curriculum 2005). Pretoria: South Africa.

Clarkson, P. C. (1991). *Bilingualism and mathematics learning.* Geelong: Deakin University Press.

Cummins, J. (1981). *Bilingualism and minority language children.* Ontario: Ontario Institute for Studies in Education.

D'Andrade, R., & Strauss, C. (Eds.). (1992). *Human motives and cultural models.* Cambridge: Cambridge University Press.

Dawe, L. (1983). Bilingualism and mathematical reasoning in English as a second language. *Educational Studies in Mathematics, 14*(1), 325-353.

Department of Education [DoE]. (1997). *Senior phase policy document*. Pretoria: Department of Education.

Durkin, K. (1991). Language in mathematical education: An introduction. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 1-3). Milton Keynes: Open University Press.

Fairclough, N. (1995). *Critical discourse analysis*. London: Longman.

Friedman, S. (1997, March 3). Democracy stifled by language barrier. *Business Day,* p. 7.

Gee, J. (1999). *An introduction to discourse analysis: Theory and method.* London: Routledge.

Granville, S., Janks, H., Joseph, M., Mphahlele, M., Ramani, E., & Watson, P. (1998). English without g(u)ilt: A position paper on language in education policy for South Africa. *Language in Education, 12*(4), 254-272.

Grosjean, F. (1982). *Life with two languages.* Cambridge, MA: Harvard University Press.

Grosjean, F. (1985). The bilingual as a competent but specific speaker-hearer. *Journal of Multilingual and Multicultural Development, 6*(6), 467-477.

Hartshorne, K. B. (1987). Language policy in Africa education in South Africa, 1910–1985, with particular reference to the issue of medium of instruction. In D. Young (Ed.), *Bridging the gap: Essays in honour of L. W. Lanham.* Pretoria: Maskew Miller Longman.

Holland, D., & Quinn, N. (Eds.). (1987). *Cultural models in language and thought.* Cambridge: Cambridge University Press.

Khisty, L. L. (1995). Making inequality: Issues of language and meaning in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema, & L. B. Abajian (Eds.), *New directions for equity in mathematics education* (pp. 279-297). Cambridge: Cambridge University Press.

Lin, A. (1996). Bilingualism or linguistic segregation? Symbolic domination, resistance and code switching in Hong Kong schools. *Linguistics and Education, 8*, 49-84.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners.* Clevedon: Multilingual Matters Ltd.

Merritt, M., Cleghorn, A., Abagi, J. O., & Bunyi, G. (1992). Socialising multilingualism: Determinants of code-switching in Kenyan primary classrooms. *Journal of Multilingual and Multicultural Development., 13*(1 & 2), 103-121.

Moschkovich, J. (1996). Learning in two languages. In L. Puig & A. Gutierezz (Eds.), *Twentieth conference of the International Group for the Psychology of Mathematics Education, Vol. 4.* (pp. 27-34). Valecia: Universitat De Valencia.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11-19.

National Centre for Curriculum and Research (2000). *Language in the classrooms: Towards a framework for intervention.* Pretoria: Department of Education (DoE).

National Department of Education. (1997). *Curriculum 2005: Lifelong learning for the 21st century.* Pretoria: NDE.

Ncedo, N., Peires, M., & Morar, T. (2002). *Code-switching revisited: The use of language in primary school science and mathematics classrooms.* Paper presented at the Tenth Annual Conference of the South African Association for Research in Mathematics, Science and Technology Education, Durban, South Africa.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* London and New York: Routledge & Kegan Paul.

Pimm, D. (1991). Communicating mathematically. In K. Durkin & B. Shire (Eds.), *Language in mathematical education* (pp. 17-23). Milton Keynes: Open University Press.

Rakgokong, L. (1994). *Language and the construction of meaning associated with division in primary mathematics.* Paper presented at the The Second Annual Meeting of the Southern African association for Research in Mathematics and Science Education, Durban.

Reagan, T. G., & Ntshoe, I. (1992). Language policy and black education in South Africa. In I. A. Stevenson (Ed.), *Sociolinguistics for applied linguistics.* Pretoria: University of Pretoria.

Rowland, T. (1995). Between the lines: The languages of mathematics. In J. Anghileri (Ed.), *Children's mathematical thinking in the primary years: Perspectives on children's learning* (pp. 54-74). London: Cassell.

Sachs, A. (1994). *Language rights in the new constitution.* Cape Town: South African Constitutional Studies Centre, University of the Western Cape.

Saunders, G. (1988). *Bilingual children: From birth to teens.* Clevedon: Multilingual Matters, Ltd.

Secada, W. (1991). Degree of bilingualism and arithmetic problem-solving in Hispanic first graders. *Elementary School Journal., 92*(2), 213-231.

Secada, W. (1992). Race, ethnicity, social class, language and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning.* (pp. 623-660). New York: Macmillan.

Setati, M. (1996). *Code-switching and mathematical meaning in a senior primary class of second language learners.* Unpublished master's thesis, University of the Witwatersrand, Johannesburg, South Africa.

Setati, M. (1998). Code-switching in a senior primary class of second language learners. *For the Learning of Mathematics, 18*(2), 114-160.

Setati, M. (2002). *Language practices in intermediate multilingual mathematics classrooms.* Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Setati, M., & Adler, J. (2001). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*(3), 243-269.

Setati, M., Adler, J., Reed, Y., & Bapoo, A. (2002). Incomplete journeys: Code-switching and other language practices in mathematics, science and English language classrooms in South Africa. *Language and Education*, 128-149.

Sfard, A., Nesher, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning mathematics through conversation: Is it good as they say? *For the Learning of Mathematics, 18*, 41-51.

Stephens, M., Waywood, A., Clarke, D., & Izard, J. (Eds.). (1993). *Communicating mathematics: Perspectives from classroom practice and current research*. Victoria: Australian Council for Educational Research.

Taylor, N., & Vinjevold, P. (1999). *Getting learning right.* Johannesburg: Joint Education Trust.

Varughese, N. A., & Glencross, M. (1996). *Mathematical language among first year university students.* Paper presented at the Fourth Annual Meeting of the Association for Research in Mathematics, Science and Technology Education, Pietersburg, South Africa.

Zepp, R. (1989). *Language and mathematics Education.* Hong Kong: API Press.

[1] Politics in this paper is used to refer to "anything and anyplace where human social interactions and relationships have implications for how social goods are or ought to be distributed. By social goods I mean anything that a group of people believes to be a source of power, status, or worth" (Gee, 1999, p. 2).

[2] In this paper the term African is used to refer to the majority indigenous population that speak African

languages. The so-called "coloureds" and Indians are thus not included in this category, since either English or Afrikaans is their main language.

[3] *Main language* refers to the language most often used by an individual, in which he or she becomes proficient. Some people who are fully bilingual or multilingual may use two or more languages on an approximately equal basis and thus have more than one main language. These people may choose to use one of their main languages in some contexts and the other main language in other contexts. I prefer to use the term "main language" and to avoid "first language" or "mother tongue".

Conferences 2003…

AAMT Australian Association of Mathematics Teachers http://www.aamt.edu.au/2003/info.html	Brisbane, Australia	Jan. 13–17
MAA-AMS Joint Meeting of the Mathematical Association of America and the American Mathematical Society http://www.ams.org/amsmtgs/2074_intro.html	Baltimore, MD	Jan. 15–18
AMTE Association of Mathematics Teacher Educators http://www.sci.sdsu.edu/CRSME/AMTE	Atlanta, GA	Jan. 30–Feb. 1
CERME 3 European Society for Research in Mathematics Education http://www.dm.unipi.it/~didattica/CERME3/Paper.htm	Bellaria, Italy	Feb. 28–Mar. 3
NCTM National Council of Teachers of Mathematics, Research Presession & Annual Conference http://www.nctm.org	San Antonio, TX	Apr. 7–12
Mα The Mathematical Association http://m-a.org.uk	Norwich, UK	Apr. 12–15
AERA American Educational Research Association http://www.aera.net/meeting	Chicago, IL	Apr. 21–25
PICME 10 Nordic Pre-conference to the 10th International Congress on Mathematical Education http://www.msi.vxu.se.picme10	Växjö University, Sweden	May 9–11
CMESG/GCEDM Canadian Mathematics Education Study Group/Annual Conference http://plato.acadiau.ca/courses/educ/reid/cmesg/cmesg.html	Acadia University, Nova Scotia, Canada	May 30–June 3
ICIAM International Congress on Industrial and Applied Mathematics http://www.iciam.org/iciamHome/iciamHome_tf.html	Sydney, Australia	July 7–11
XI CIAEM Eleventh Interamerican Conference on Mathematical Education http://www.furb.br/xi-ciaem	Blumenau-SC, Brazil	July 13–17
PME and PME-NA Joint International and North American Conference on the Psychology of Mathematics Education http://www.hawaii.edu/pme27	Honolulu, HI	July 13–18
JSM of the ASA Joint Statistical Meetings of the American Statistical Association http://www.amstat.org/meetings	San Francisco, CA	Aug. 3–7
GCTM Georgia Council of Teachers of Mathematics http://www.gctm.org	Eatonton, GA	Oct. 16–18
SSMA School Science and Mathematics Association http://www.ssma.org	Columbus, OH	Oct. 23–25

Coping with New Mathematics Teacher Roles in a Contradictory Context of Curriculum Change

Mellony Graven

This paper is part of a broader longitudinal study that investigates, from a social practice perspective, mathematics teacher learning (within an in-service program) stimulated by rapid curriculum transformation (Graven, 2002). As a backdrop for the description of the curriculum that follows, I begin with a discussion of the socio-political context that gives rise to the new curriculum. Then I provide a sketch of the wider study of which this analysis forms part, and so situate my focus on teachers' roles within a theoretical and methodological framework. Documentary analysis allows me to describe the curriculum and the new teacher roles. I conclude with a discussion of conflicts and tensions that arise in relation to these roles in curriculum implementation.

The social and political context within which the study takes place

South Africa has been typified by large inequalities. Wilson & Ramphele (1989) note that of the 57 countries for which data is available, South Africa displayed the widest gaps between rich and poor. The system of apartheid was predicated on ensuring that these inequalities were structured along racial lines. Under apartheid four racially classified population groups were created: White (of European origin); Colored (of mixed race, mainly European, African, and Malaysian); Asian (of Asian origin); and African (of African origin). All South Africans not designated as White were denied democratic participation, and resources were allocated to groups differentially for education, health, and all other essential services. Thus, huge inequalities were created and perpetuated under apartheid, resulting in large gaps between the rich and largely white population, and the poor and largely black population.

The education system under apartheid consisted of racially segregated departments of education. Thus all government-funded schools were racially segregated. Schools were hierarchical institutions with a culture of top-down decision-making and passive acceptance of instructions by teachers. Teaching in schools primarily involved the delivery of a prescribed, centralized curriculum that was subject to inspection. Teaching was dominated by teacher-centered "chalk and talk" methods, and assessment was almost synonymous with tests and examinations (Graven, 2002).

Since the first democratic elections in 1994, South Africa has been embarking on radical educational reform. The need for a complete overhaul of the education system under apartheid has been identified as a priority for building a new democratic South Africa. Thus educational change has been stimulated by the major political changes which occurred in the country during the 1990s and which brought about the abolition of apartheid and the introduction of a democratic South Africa. The vision for education that emerged was to integrate education and training into a system of lifelong learning. Outcomes-based education (OBE) was adopted as the approach that would enable the articulation between education and training, recognition of prior learning, and thus increased mobility for learners between different vocations.

Through consultation with a range of stakeholders, including teachers, a new curriculum, *Curriculum 2005* (National Department of Education [NDE], 1997), was developed for implementation. However, the degree of teacher involvement with the project has been criticized, particularly with regard to the number of teachers who participated in the curriculum's design, the demographics of those teachers who were involved, and the extent of teachers' participation (Jansen & Christie, 1999).

Curriculum 2005 is premised on a learner-centered, outcomes-based approach to education. The key principles on which *Curriculum 2005* is based are: integration, holistic development, relevance, participation and ownership, accountability and transparency, learner-orientation, flexibility, critical and creative thinking, progression, anti-biased approach, inclusion of learners with special education needs, quality standards, and international comparability (NDE, 1997). It should be noted that these changes in education did not originate in *Curriculum 2005*. South Africa has a long history of attempts to introduce "alternative curricula," most notably the People's Education movement and the National Education Co-ordinating Committee.[1] Chisholm, et al. (2000) sum up this history: During the apartheid years the principal pedagogical alternative to

Mellony Graven has recently completed her Ph.D. at the University of the Witwatersrand, Johannesburg, South Africa. She is currently working at the university in mathematics teacher education. Her research interests are investigating mathematics teacher learning from a socio-cultural perspective.

the education system's Fundamental Pedagogics was "progressive education", a form of learner-centered education nurtured in the liberal universities and the English private schools. In the 1980s the progressive learner-centered approach was linked to an egalitarian transformative project for South African education, and the result, People's Education, was presented as the alternative to "apartheid education" (p. 26).

However, the effectiveness of these attempts was muted, and the curriculum of the apartheid state clearly dominated (Jansen, 1999). Changes in the political landscape opened the way for progressive education stakeholders to become involved in policy debates about the future of education. The main features of People's Education that were absorbed into contemporary policy were equal access for all, critical thinking, learner-centeredness, bridging the gap between theoretical and practical knowledge, teachers as curriculum developers, group work, community participation, and continuous assessment (Chisholm, et al., 2000).

A distinguishing feature of *Curriculum 2005* is its explicit political agenda. *Curriculum 2005* is a vehicle for restructuring South African society along democratic principles. This role is captured in the introduction to the *Curriculum 2005*:

> The curriculum is at the heart of the education and training system. In the past the curriculum has perpetuated race, class, gender and ethnic divisions and has emphasised separateness, rather than common citizenship and nationhood. It is therefore imperative that the curriculum be restructured to reflect the values and principles of our new democratic society (NDE, 1997).

These underlying goals of *Curriculum 2005* have taken shape in the new mathematics curriculum and in its demands for new teacher roles. Before describing these changes, I discuss briefly the broader study from which this paper is drawn and why an analysis of curriculum change from the perspective of teacher roles and identity is important.

The empirical field of the study

The context of curriculum change implies an important role for in-service work with teachers. The Programme for Leader Educators in Senior-phase Mathematics Education (PLESME) was developed in order to create leader teachers in mathematics with the capacity to interpret, critique, and implement current curriculum innovations in mathematics education in South Africa. Other major aims included:

- enabling and fostering collegial and co-operative ways of working with other mathematics teachers within schools and between schools;
- fostering co-operative ways of working with departmental mathematics subject advisors and district offices to assist in implementing and reviewing mathematics curriculum innovations;
- developing necessary skills and knowledge for running workshops with groups of teachers on a range of mathematics topics related to current curriculum innovations.

Assessment was portfolio-based. Portfolios included, for example, teacher conference presentations, materials and booklets designed by teachers, teachers' input into the Report of the Review Committee on *Curriculum 2005* (Chisholm, et al., 2000), workshops teachers organized and conducted, classroom videos, and teachers' written reflections on lessons, etc. PLESME worked with teachers from schools in Soweto and Eldorado Park (both urban townships outside Johannesburg) over a two-year period. This in-service teacher education program provided the empirical field for my study.

In PLESME I wore two hats. First, I was the coordinator of PLESME, my full time vocation from October 1998 until June 2001. I raised funds for it; designed it; set up a steering committee; and negotiated with schools, districts and teachers as to the nature of the project. I was accountable to my organization, the university, the steering committee, donors, teachers, and schools for the value and "success" of the project. At the same time, I was a researcher in the process of conducting research on the nature of mathematics teacher learning in relation to an in-service teacher education within the context of rapid curriculum change.

I was expecting some tension to emerge in relation to my role as an in-service education coordinator and my role as researcher, primarily because I had struggled to distinguish these roles clearly in the research proposal. However, I discovered that no such tension emerged in practice; the tension remained primarily theoretical. Instead I discovered a powerful praxis in the duality of performing both roles. It enhanced and enabled a form of action-reflection practice that I had been unable to achieve with success in previous in-service teacher education projects. For example, reflecting on interviews, lessons, and other data helped me to develop research ideas and refine my research objectives. This reflection led to asking specific questions in interviews and questionnaires that related to my research interest in understanding the nature of teacher learning. However, such reflection on data also led to the re-planning of PLESME activities and the design of additional activities that enhanced teacher participation and teacher learning. For example, interviews became discussions that formed a necessary part of praxis and were also geared towards gathering data necessary to assist me in answering my research questions. Similarly, my ongoing reflection in the form of journal entries (relating both to PLESME and my work as a researcher) and the readings I was engaged with helped me reflect on how to improve PLESME.

Teacher learning, roles and identity

The study explored mathematics teacher learning in relation to how teachers participate in and make use of a community of practice, stimulated by PLESME in the context of curriculum change. The study is broadly located in social practice theory. Within this field, Lave & Wenger's (1991) notion of participation in communities of practice is becoming increasingly popular to explain learning. According to their model, learning is located in the process of co-participation, the increased access of learners to participation, and in an interactive process in which learners simultaneously perform several roles. Participation in this sense is the process of "being active participants in the practices of social communities and constructing identities in relation to these communities" (Wenger, 1998, p. 4). Learning and a sense of identity are aspects of the same phenomenon (Lave & Wenger, 1991). Previous research conducted by Graven (1998) indicated that teacher education should involve bringing teachers into supportive communities where reflection-in-practice is enabled. Lave and Wenger's model of learning supported this conclusion and provided some useful insights for analysis of the broader study.

My assumption was that the implementation of the new curriculum would not simply involve following a set of curriculum instructions or replacing "old" practice with "new" practice. Rather, implementation is a process of fashioning the curriculum in such a way that it becomes part of the teacher's "way of being." In fashioning the curriculum in this way, teachers will change themselves and modify the curriculum. My assumption was that this learning would take place within the context of participation within the PLESME practice, which included practice within schools. These assumptions were not evident to me at the start of the research study but rather developed over time through observing teachers make sense of the new curriculum and reflect on their learning process. In interviews with teachers about their learning within the context of PLESME, it became evident that teachers themselves saw their learning as a process of developing a different way of being. The following quotes from teacher interviews support this statement.

Beatrice,[2] a grade 7 primary school teacher, said, "You know before I always used to introduce myself as the music teacher, now I introduce myself as the maths teacher" (Beatrice, personal communication, July 20, 1999). Through learning and being part of a mathematics community, this teacher's identity as a mathematics teacher was strengthened.

Elaine, another teacher in the study, said "It [PLESME] has broadened my horizons very much…For myself, if I open a newspaper I think what can I use in my class, or think this is another way of drawing a graph... Like the example we did on holiday, I start to realise how much they (advertisements) are bluffing you. I use it in everyday life…" (Elaine, personal communication, June 22, 1999). For Elaine, participation in PLESME practices led to a new mathematical approach to the world around her—she became a critical mathematical thinker in her life outside of the mathematics classroom.

Two key notions I draw upon are teacher roles (designed by the NDE) and teacher identities (which form in uneven ways in relation to change). The object of the broader study is to elaborate on the relationship between these notions. I believe that analysis of curriculum change from the perspective of teacher roles and identities is original and has much to contribute to understanding curriculum in practice.

The study uses ethnography as its research methodology, in which I work as a participant observer. Because teacher learning is analysed within the context of radical curriculum change, a major part of the study has involved thorough documentary analysis of the new curriculum and related literature. This part of the study is the focus of this paper. For a more detailed analysis see Graven (2001). I have drawn on the work of Bernstein (1982, 1996) for tools for curriculum analysis. In this paper I draw on Bernstein's (1996) differentiation between performance- and competence-based pedagogic models. According to Bernstein, performance models serve primarily economic goals and are considered instrumental. They emphasize specialized skills necessary for the production of specific outputs. In contrast, competence models foreground the cognitive and the social, and acquirers apparently have a greater measure of control over selection, sequence, and pace. I also draw on Bernstein's concept of *Official Projected Identities*, which refers to the identity projected by an institution (in this case, the NDE).

Changes in the mathematics curriculum and teacher roles

In this section I describe the changes found in mathematics curriculum documentation and unpack the new roles for teachers. First, in *Curriculum 2005* the subject *Mathematics* has been replaced with the broader Learning Area *Mathematical Literacy,* *Mathematics and Mathematical Sciences* (MLMMS). This learning area represents a major shift in the philosophy of mathematics and mathematics education. Three main philosophical shifts can be identified. They relate to the approach to mathematics teaching, the nature and contents of mathematics, and the role of mathematics education. I will address each of these changes briefly.

Within the learning area MLMMS, the NDE defines mathematics as:

> the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction (1997, p. 2).

This definition places an emphasis on more social constructivist, learner-centered, and integrated approaches to mathematics teaching and learning. This emphasis indicates a move away from the previous performance-based approach towards a more competence-based approach. Furthermore this definition indicates a shift away from the "absolutist paradigm," which views mathematics as a body of infallible objective truth that has little to do with the affairs of humanity (Ernest, 1991). The *Rationale* for MLMMS further states that mathematics should empower learners to "understand the contested nature of mathematical knowledge" (NDE, 1997, p. 1). MLMMS focuses its attention on constructing mathematical meaning in order to understand the world and make use of that understanding. Mathematical learning is to be relational, flexible, transferable, and integrated with everyday life and other learning areas. The specific outcomes for MLMMS indicate changes in the content of school mathematics. The importance of data, space, and shape (not simply Euclidean geometry); history of mathematics; and cultural, social, and political applications of mathematics are all new. For example, Specific Outcome 4 is: "Critically analyze how mathematical relationships are used in social, political and economic relations" (p. 3).

The specific outcomes support the important role charged to MLMMS for helping to build a new democratic, equitable, non-racist, non-sexist South Africa. Political aims are also clear in the *Rationale* for MLMMS, which states that MLMMS must empower people to:

- work towards the reconstruction and development of South African society;
- develop equal opportunities and choice;
- contribute towards the widest development of the society's cultures;
- participate in their communities and in the South African society as a whole in a democratic, non-racist and non-sexist manner etc.

In sum, MLMMS demands major philosophical shifts of teachers and learners. These shifts affect teacher roles and hence the development of mathematics teacher identities. As is well documented (Thompson, 1992), bringing about change in teachers' conceptions of mathematics is a difficult and long-term process. Therefore it is important not to underestimate the enormity of these demands.

Further analysis of MLMMS shows four different orientations of mathematics.

1. Mathematics is to be learned for critical democratic citizenship. It empowers learners to critique mathematical applications in various social, political, and economic contexts.
2. Mathematics is relevant and practical. It has utilitarian value and can be applied to many aspects of everyday life.
3. Mathematics inducts learners into what it means to be a mathematician, to think mathematically, and to view the world through a mathematical lens.
4. Mathematics involves conventions, skills, and algorithms that must be learned. Many will not be used in everyday life but are important for further studies.

An understanding of school mathematics, in terms of the four orientations, demands that mathematics teachers develop related "roles" in relation to their teaching practice. Four related mathematics teacher roles are thus identified:

1. The teacher's role is to prepare learners for critical democratic citizenship. The teacher becomes a critical analyzer of the way mathematics is used socially, politically, and economically, and supports learners to do the same.
2. The teacher's role is a local curriculum developer and an applier of math in everyday life. The teacher brings math from "outside" into the class.
3. The teacher's role is to be an exemplar mathematician or someone who has an interest in pursuing mathematics for its own sake. The teacher apprentices learners into ways of investigating mathematics.
4. The teacher's role is as a custodian of mathematical knowledge or a deliverer of mathematical conventions, algorithms, etc., which are important for MLMMS in general and will enable success in the Further Education and Training band (grades 10-12). The teacher is a conveyor of the practices of the broader community of mathematics teachers.

In this vision for change, it's important to ask whether these roles are realizable. Is it possible for teachers to perform each of these mathematical roles? Is it reasonable to expect teachers to integrate across these roles? Engaging in a theoretical discussion about these issues is beyond the scope of this paper. Instead I examine some of the tensions that emerge in relation to these roles in the implementation of the new curriculum.

Some tensions in working with the mathematics orientations and teacher roles

The separate presentation of the four orientations and related roles should not indicate a lack of connection between them; these orientations should work together in support of each other. While the assumption in MLMMS is that these orientations can and do co-exist, they are not presented to teachers this way in practice. Rather than presenting a view of mathematics that integrates all four of these orientations and roles, emphasizing variation, curriculum support presents conflicting messages as to which orientation is "best," and often sends a message that there is one best orientation a teacher should adopt. Official support for primary school mathematics teachers at district level tends to focus on the first and second orientations while viewing the fourth orientation, the one most familiar to teachers, as "old." On the other hand, support provided to teachers that is aimed at improving performance in mathematics examination results emphasizes the fourth orientation at the expense of the other three. Let me elaborate with two examples.

Illustrative Learning Programmes (ILPs) were designed by the Gauteng Department of Education and the Gauteng Institute for Curriculum Development to support teachers in developing theme-based and integrated learning materials (1999). The first ILP for MLMMS, grade 7, was "Farming and Growth." Analysis of this 50-page document reveals that only approximately one quarter of the activities relates to mathematics and that most of these mathematics activities simply "apply" mathematics skills that are assumed to be available to learners. The mathematics in this ILP works with the second orientation at the expense of the other three orientations. This ILP has been heavily criticized by mathematics teachers and educators. Minutes of the Primary Mathematics Working Group Session of the Association of Mathematics Education of South Africa (2000) reflect that teachers feel that there is not enough mathematics in this ILP. Chisolm, et al. (2000) note that the ILP shows that the emphasis on integration has compromised coherent mathematical development and that the mathematical content is obscured.

On the other hand, official support aimed at the improvement of performance emphasizes the fourth orientation by stressing algorithms, procedures, and definitions. At the start of my work with the PLESME teachers I was invited to a district level workshop for primary school teachers from Soweto. These teachers were invited to a previously white primary school for the workshop. At this workshop the white teachers from this school provided the black teachers from Soweto with photocopies of their mathematics schemes of work. These schemes of work did not reflect any current curriculum developments and only focused on the fourth orientation of mathematics. The common assessments given to the teachers from Soweto schools were based on this scheme of work and did not reflect any of the other three orientations. For example, the exam asked learners to define various mathematics terms and excluded geometry because according to the scheme of work, this mathematical topic is only dealt with in the final term. The justification for the insistence of the use of these schemes of work and assessments is that they are derived from a so-called "top performing" school in the district. This judgment of top-performance was based on the grade 12 external exit assessment of learners in the high school that this primary school fed into (Researcher's journal entry, February, 1999).

Recall that under apartheid white schools were provided far greater resources than black schools. As a result, performance on grade 12 examinations for white schools was far better than for black schools. Clearly the district advisor who organized the workshop (himself an former teacher from Soweto) assumed that good results in grade 12 meant that "good" teaching must have occurred at the primary school level. Therefore he assumed that black teachers (in Soweto) should learn from the white teachers irrespective of whether or not they embraced the new curriculum and its socio-political aims.

Such actions by the part of district workers will affect the morale of teachers, undermining teachers' attempts to implement new curriculum ideas and excluding teachers from making decisions related to the teaching and assessment of their learning area. Furthermore, they will prevent, rather than support, teachers from developing new roles that resonate with MLMMS and broader curriculum changes. In a context of a post-apartheid South Africa, the racial undertones of such an incident which imply that learning between teachers of different race groups is a one-way process from "previously advantaged Whites" to "previously disadvantaged Blacks," are particularly problematic and worrying.

Thus two contradictory official identities are being projected, that of the incoming curriculum and that of the outgoing curriculum. The Official Projected Identity (Bernstein, 1996) of MLMMS, the incoming curriculum, emphasizes the first and second orientation (the third and fourth orientations are included but are, in practice, less emphasized). However, the Official Projected Identity related to the outgoing (but still predominantly implemented) curriculum emphasizes the fourth orientation. Since there are currently two curricula existing within the school system, the incoming competence-based model and the outgoing performance based-model, provincial departments and district workers are in the difficult position of having to work out when it is appropriate to work with which Official Projected Identity. Furthermore, though *Curriculum 2005* applies to all bands of education, including Early Child Development, General Education and Training (GET, grades 1-9) and Further Education and Training (FET, grades 10-12), currently details of the curriculum are only available in the GET band. Since the curriculum has not yet been designed for the FET band, the credibility of the first and second orientations is undermined. The alternating official emphases on the two different curricula create a swinging pendulum in which teachers receive contradictory messages. I believe that these inconsistent emphases are problematic and that all four orientations are needed for learners to become competent in MLMMS.

I have argued that analysis of curriculum documentation for MLMMS reveals a radical shift in the philosophy of mathematics. Furthermore, during the phasing-in period of *Curriculum 2005*, two contradictory education models "officially" co-exist. This duality creates dilemmas for teachers who are expected to implement new learner-centered and locally relevant curricula while their schools continue to be judged on the performance of national examination results. I believe that this tension is reflective of broader tensions between the local and global. *Curriculum 2005* attempts to satisfy both local and global demands in its drive to create mathematical meaning in local contexts while simultaneously competing internationally.

Wenger (1998) raises an important issue for teacher education in this respect. While national education departments can design roles, they cannot design the (local) identities of teachers. The broader research study analyzes teacher learning in terms of the relationship between the new mathematics roles, the generic roles for educators as outlined in the *Norms and Standards Document for Educators* (NDE, 2000), and developing teacher identities. In this paper I have outlined the socio-political context that has led to the design of new teacher roles that in turn have resulted in contradictory messages for teachers. I have used evidence from the larger study as examples of these contradictions.

In conclusion, I concur with Harley and Parker (1999) that teacher development in this context of change is far more complex than simply retraining teachers. Ways must be found to support teachers in developing new professional identities. They conclude that to implement these changes "teachers may well need first to shift their own identities, their understanding of who they are and how they relate to others" (p. 197).

REFERENCES

Association of Mathematics Education of South Africa. (2000). *Minutes of the Primary Mathematics Working Group Session.* Bloemfontein, South Africa.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique*. London: Taylor and Frances.

Bernstein, B. (1982). On the classification and framing of educational knowledge. In T. Horton & P. Raggatt (Eds.), *Challenge and change in the curriculum.* Milton Keynes, UK: The Open University.

Chisholm, L., Volmink, J., Ndhlovu, T., Potenza, E., Mahomed, H., Muller, J., Lubisi, C., Vinjevold, P., Ngozi, L., Malan, B., & Mphahlele, L. (31, May, 2000). *A South African curriculum for the twenty-first century.* (Report of the review committee on Curriculum 2005). Pretoria: South Africa.

Ernest, P. (1991). *The philosophy of mathematics education: Studies in mathematics education*. London: The Falmer Press.

Gauteng Department of Education (GDE) & Gauteng Institute for Curriculum Development (GICD). (1999). *MLMMS grade 7 module 1: Farming and growth, learner's material.* Johannesburg: GICD.

Graven, M. (1998). Are mathematics high school teachers ready for Curriculum 2005? In *Proceedings of the 6th Annual Meeting of the Southern African Association For Research in Mathematics and Science Education, Vol. 1* (pp. 172-178). Pretoria, South Africa.

Graven, M. (2001). Understanding the context of South African curriculum change. In *Proceedings of the 9th Annual Meeting of the Southern African Association for Research in Mathematics and Science Education, Vol. 1* (pp. 327-35). Maputo, Mozambique.

Graven, M. (2002). *Mathematics teacher learning, communities of practice and the centrality of confidence.* Unpublished doctoral dissertation, University of the Witwatersrand, Johannesburg South Africa.

Harley, K., & Parker, B. (1999). Integrating differences: Implications of an outcomes-based national qualifications framework for the roles and competencies of teachers. In J. Jansen & P. Christie (Eds.), *Changing curriculum: Studies on outcomes-based education in South Africa* (pp. 181-200). Kenwyn, South Africa: Juta & Co.

Jansen, J. (1999). Why outcomes-based education will fail: An elaboration. In J. Jansen & P. Christie (Eds.), *Changing* *curriculum: Studies on outcomes-based education in South Africa.* (pp. 145-156). Kenwyn, South Africa: Juta & Co.

Jansen, J., & Christie, P. (Eds.). (1999). *Changing curriculum: Studies on outcomes-based education in South Africa*. Kenwyn, South Africa: Juta & Co.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* New York: Cambridge University Press.

National Department of Education. (1997). *Curriculum 2005: Lifelong learning for the 21st century*. Pretoria: NDE.

National Department of Education. (4 February 2000). *Norms and standards for educators* (Government Gazette, Vol. 415, No. 20844). Republic of South Africa: NDE.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* New York: Cambridge University Press.

Wilson, F., & Ramphele, M. (1989). *Uprooting poverty: The South African challenge.* Cape Town, South Africa: David Phillip Publishers.

[1] The NECC was an alliance of progressive education and labor stakeholders. In 1992 its Curriculum Research Group produced a National Education Policy Investigation report on curriculum on which much of the current curriculum is based.

[2] All names used are pseudonyms.

Mathematicians' Religious Affiliations and Professional Practices: The Case of Charles

Anderson Norton III

This paper reports on the second of three case studies, all intended to explore the implications of religious affiliation in the professional lives of mathematicians. These case studies yield implications for various topics within the field of mathematics education. For example, each of the first two cases has revealed a religious influence on the participant's childhood decision to study mathematics. Naturally, we might conclude that such an influence exists for many school-aged, religious mathematics students. Other implications range from the mutual influence of students' mathematical and religious practices to the religious value of teaching and researching mathematics. In this spirit, I report on my experiences with Charles, the second of the three mathematicians of my study.

One might find religious implications for various professions, but professional mathematics provides particularly interesting cases: Mathematics, as a discipline, has a long reputation for providing truth and certainty. Though more recently this reputation has been called into question (Kline, 1980), there is something about the context-free, abstract nature of mathematics that makes the subject seem incontrovertible. Yet religion is often considered as an avenue to Truth. In fact, I recall one of my undergraduate mathematics professors proclaiming that "mathematics is the only truth with the possible exception of theology." Well then, how might these two truths co-exist?

In a previous paper (Norton, 2002), I reported on the first of the three case studies concerning the relationship between mathematicians' religious beliefs and professional practices. From my experiences with that participant—a Jewish man named Joseph—I concluded that mathematicians must reconcile their practices with their life philosophies or religions in order to make their mathematical practice meaningful. This reconciliation is difficult when mathematical thought and religious beliefs (and values) are viewed as contradictory. In fact, such a view is the case for Charles.

"If the scientific community concedes even one miraculous event, then how can it credibly contest the view that the world (and all its fossilized relics) was created in one instant just 6,000 years ago?" (Singham, 2000, p. 428). Singham's short statement summarizes the ongoing conflict between religious belief (especially Judeo-Christian beliefs) and scientific thought. Nord's reply to such questions, on the other hand, anticipates one possible resolution by noting that evolution and other scientifically defined processes may just be "God's way of doing things" (1999, p. 30). The purpose of this paper is to analyze the similar conflict and resolution experienced by Charles so that we might draw conclusions for mathematics education from his struggle. Indeed, an emergent theme from this case—the paucity of value for secular study (and, indeed the devaluation of many scientific branches that seem to contradict Biblical truth)—may have important implications for the work of mathematics teachers in secondary schools in the United States.

Methods

In order to study the implications of religious affiliations in the lives of professional mathematicians, I conducted interviews with three university mathematics professors. I identified three religious groups representing the diversity of religious beliefs in their mathematics department: Jewish, Christian, and Buddhist. Here I will abbreviate my report on the methods of the larger study, which can be found in Norton (2002), and focus on the case of Charles. Like myself, Charles is a Christian but our views are somewhat different because I am a Catholic and he is a Protestant.

Charles is a full professor in a large southern university's mathematics department and is expected to do mathematical research and teach classes. However, he also has a long list of additional duties that are described in the background section. Data collection for Charles' case was similar to the other cases. I conducted a single, one-hour interview and was able to collect additional data from archival sources. These documents included his online vita and a booklet describing the faculty of their department. I used this data in addition to some of the interview data for background information about Charles.

After transcribing, reading, and rereading the interview transcript, I coded, grouped, and identified concepts from the data. These concepts were then developed in narrative form. First I developed

Andy Norton is currently working on his doctoral dissertation in mathematics education and master's degree in mathematics at University of Georgia. His research interests include students' mathematical conjectures and their role in learning.

paragraphs from the concepts. Then I identified the major themes relating the participant's religion and profession and restructured the narrative section around these themes. The final narrative is the central component of the analysis of Charles' case. In order to relate Charles' case and build on emergent themes, I recount a few histories in the discussion section that I will use as a backdrop

Since the narratives are made up of my own words, I wanted to include something additional to capture Charles' words and phrases. So I incorporated poetic transcription (Glesne, 1999), restructuring words from the transcripts into a poem. I began with a list of phrases and organized them into stanzas centered on particular themes or concepts. The stanzas began to take form as I shuffled and reduced quotes. In forming the stanzas, I was careful to stay close to my interpretations of their meaning. While I used only Charles' literal phrases and words in this section, their order and concatenation may be very different from the literal transcriptions. I hope that the end result gives the flavor of the participant's voice and language that is missing from the narratives. Thus my presentation of Charles' story consists of three parts: background, narrative, and poem.

Charles' story

Background

Charles is a European-American man, about fifty years of age. He was raised in his mother's church, the United Church of Christ, in Montana. His mother and two sisters were very devoted Christians, but his father was agnostic. Charles struggled with many of the Christian doctrines, such as the deity of Jesus, for much of his life. The tension between him and his sisters led him to renew his faith during graduate school, but he continued to wrestle with many church doctrines years later.

Charles was never a very social person. Even as a child his extracurricular activities were restricted to band and church. At a very young age Charles also became aware of his mathematical talent. He was set apart from his peers in public school mathematics classes, often working alone or with a small group of other gifted students on higher-level mathematics. His mathematical talent also caught the attention of his professors in college and eventually led to his graduate studies at Princeton.

Charles is now married and a father of three. He describes his profession as one of teaching, advising, serving on committees, and helping to make departmental decisions. His research (mostly in Number Theory) must be squeezed in whenever other commitments taper off, such as at the beginning of the school year. This situation is very different from the one he imagined for himself when he decided to become a mathematics professor. Though he seems to enjoy teaching, research is his main interest and he considers many other duties subsidiary to that.

Narrative

Struggle followed by peace. In describing his religious beliefs and mathematical interests, Charles talked a lot about his childhood and the frequent conflict he experienced in his family. He described his mother as a very strong person who worked in the church. In fact, his parents both signed the original covenant of their Congregationalist church in Montana. However, Charles' father perceived contradiction between Biblical inerrancy and theories of evolution, which made religion problematic for him. These perceptions led to quarreling between Charles' parents, quarreling that ended when the father stopped attending church. The theme of conflict followed by peace continues throughout Charles' story.

Charles' two sisters were both very involved with Protestant Christian churches. Both went on religious missions, and one became a pastor. However, their church was not the same as their mother's. This difference led to tensions between Charles' mother and sisters. Eventually, his mother came to peace with his sisters' decisions, but his sisters' strong faiths continued to cause tension for Charles who, in contrast, had not become comfortable with his Christianity.

Throughout his life Charles has attended church regularly, though he has struggled with many doctrinal issues. In particular, he seemed to share his father's view that creationism is inferior to evolutionism. As a high school class assignment, he wrote a rebuttal to the theory of creationism; his sisters had written in favor of it in similar assignments. The tension between his scientific views and his sisters' faith in religious doctrine remained throughout Charles' graduate school studies.

Charles noted that while growing up he was not part of a church youth group and that he had been quite shy. Although he had attended church through graduate school, it wasn't until he began post-doctoral work in Cambridge that he found a group of young Christians with which he could identify. At that time, he renewed his own Christian beliefs. He said that it was the tension with his sisters that brought him to the point of renewal. Though he continued to struggle with many other doctrinal issues, he founded his beliefs on three main doctrines: "I believed that God answered prayer… that if Jesus were alive I would follow him… and that I couldn't be justified before God on my own merits."

When he moved to the South in 1981, Charles began attending a Presbyterian Church. There, a friend questioned him about the doctrine of Jesus' deity, and Charles resisted the provision of pat answers to these questions. "I wasn't going to be steam-rolled into any doctrinal confessions at the start without thinking about things," he said. The tension that ensued between him and his friend led to a distancing between them. Only years later, after meeting and marrying his Christian wife, did Charles come to a peace about that issue and other religious beliefs. He had needed time to resolve such issues for himself.

During the period of his life that he was struggling with doctrinal issues, Charles was trying to reconcile his mathematical interests with his Christianity. As early as seventh grade, Charles knew that he was gifted in mathematics. His teachers knew it too. He was the best mathematics student his college professors had seen at their school. He liked mathematics because he was good at it and he enjoyed the competitiveness involved in it. At the same time, he was careful "not to show out", though he was "inwardly very proud." While he had a great deal of mathematical talent and a strong desire to develop that talent, Charles felt he needed to find religious meaning for pursuing such a profession.

When Charles was about eight years old, he prayed for his sick parakeet to get better, promising that in return he would find the best way to serve God. The parakeet got better and ever since Charles struggled with finding the best way to serve. By the end of his undergraduate years, he was "in a knot" trying to decide what career he should pursue to serve God. Though he was never gifted socially, for a time Charles thought about becoming a pastor. "I used to think that being a pastor was the only thing you could do [to serve God]…but I can see that I am not gifted to do that kind of work." He was clearly gifted in mathematics, but felt he needed to do something that would directly benefit man. He considered professions in physics or engineering, tackling environmental problems. However, upon graduating, he chose to continue doing the work he enjoyed; he began a graduate program in pure mathematics at Princeton.

Once again, Charles came to peace—this time about his choice of careers: "It's okay to have been a mathematician." With more mature judgment, he sees that everyone plays a part in God's plan. He cannot expect to produce the key idea in solving pollution problems or any other social or environmental problem. People work one step at a time on small, technical aspects of problems. This is just as true in mathematics.

God orchestrates. Charles believes that "God orchestrates everything that happens in history." This belief resolves the conflict between evolution and creationism because, as Charles explains, God created the world through evolution. The industrial revolution, evolution and other scientific developments are part of God's plan. He works through people so that they find Truth. However, "it takes the eyes of faith… to see God's hand [in it]."

As for Biblical inerrancy, Charles does not believe that God wrote the Bible, but that God inspired the authors. He feels that God was present to Isaiah, Paul, and the other Christian prophets. He reveres them as "the greatest souls that ever were", and respects them as the "giants of another domain." Because of their importance in that domain, Charles compares them to Newton and Gauss of mathematics.

God orchestrates ideas in the domain of mathematics, just as he orchestrates everything else. In all of the sciences, mankind is "wavering toward a truth." Though individual theories may fail, better ones replace them. So though people sometimes take the wrong path in their theories, there is a general trend toward Truth. Every piece of mathematical knowledge contributes to that Truth as well. Charles feels like an explorer in his own search for mathematical knowledge.

In a way, mathematics actually stands out from all other scientific knowledge: "Mathematics is the most certain of all of the sciences." Charles seems bothered by the fact that, historically, there has been a lot of vagueness in mathematics: "People would just do things [in mathematics] because they worked." Since then, people have tried to re-establish solid grounding for mathematics. There are still problems such as the existence of undecidable statements, but Charles says that shouldn't stop one from working on them. Historically, new developments shed light on problems so that they are resolved in new ways. This process is part of approaching Truth.

Looking back, Charles feels at peace with his decision to pursue mathematics and feels that God has blessed his career. He feels he is a channel used by God to bring mathematical knowledge to the world. In fact, Charles can recall at least four instances when that channel was quite direct. Each time, he was completely stuck on a mathematical problem. Each time, he prayed for an idea, and each time God gave him one. Though others may argue the idea would have come anyway, the certainty and immediacy of the ideas have made Charles believe his prayers were answered.

At the time he decided to become a mathematician, Charles anticipated a career centered on research, developing new mathematics. However, he finds himself occupied with a lot of busy work. There are committee meetings, departmental duties, and subsidiary tasks such as grading papers and meeting with students. His mathematical research must be "squeezed around the corners," when the pressure of seeing students is not so great. While he would like to focus more on his research, Charles does try to build relationships with his students as well.

In the classroom, Charles identifies himself as a Christian on the first day of each semester. He feels that this openness has had a positive influence on many of his students, though any more mention of it in the classroom might be "inappropriate." Students often approach him after class that first day to let him know that they appreciate his openness about his Christianity, and as a result some have developed stronger personal and professional relationships with him. "I've had impact on a few students—not very many," he concluded.

Wavering Toward a Truth

It takes the eyes of faith to see God's hand;
I'm probably not as conscious of it as I should be.

Church was part of her life, all of her life, but biblical truth
Was his tremendous stumbling block.
So my parents quarreled constantly, until the break point.

My sisters were youth with a mission, off in some crazy left field.
My tension, my struggle, my mother's heart anxiety,
We eventually became at peace with it.

Montana, Boston, Princeton, Georgia. Straight as an arrow,
Easily miles beyond the closest of my classmates, I kept my pride
Hidden (secret, inward, non-godly motives) and continued on a reasonable path.

I'm not going to be steam-rolled into any doctrinal confessions—not at the start,
Not without thinking about things. But if Jesus were alive now I would trust Him.
And eventually I came to a peace about the deity of Christ.

The ongoing enterprise of Mathematics—I see that as my calling
My parakeet got sick. I prayed. My parakeet got better.
I was just in a knot, but would serve God the best way I could.

Should we do this? Should we do that? Time, time, very busy, very busy time:
You get 30 unhappy undergraduates beating down your door,
And research gets squeezed in the corners of whatever time is left.

It's okay to have been a mathematician: explorer of non-physical world.
You can see this rock up ahead of you. It's not like reaching into fog.
You reach up for it, and in the fullness of time Truth will be found.

They say the universe is contracting; the next day it's expanding.
Science goes in fads (and pastors decry it as the work of the devil).
Now they think there's lots of dark matter. So we bumble along, but truth will be found.

You can either put up or shut up, you can take it as I *do* (I think it's rather unique):
I prayed for an idea, God cared about that piece of work, and
An idea came into the world. The idea came into the world.

Seeing the immense amount of vagueness, what can one person do?
One small step at a time, you shouldn't give up on the restoration of rigor.
And what surfaces at the end—that's God's.

Discussion

What can mathematics educators learn from the case of Charles? Charles' approach to mathematical meaning lies at the heart of the answer. In order to frame his approach and final stance on mathematical meaning, I begin by placing him within the historical spectrum on views of mathematical truth. Situating him historically is important because Charles' views of mathematical truth were eventually embedded in religious truth, and this larger truth gives meaning to his practice. Next, in order to highlight the void that Charles was attempting to fill, Charles' search for meaning can be compared to Joseph's built-in meaning for mathematics. Finally, I draw on Charles' search and resolution to reveal implications for mathematics classrooms. In particular, mathematics educators need to demonstrate the usefulness of mathematics in solving important social problems and invoke students' natural curiosities in the classroom so that students are motivated to develop meaning for mathematics.

Working Toward Reconciliation

Charlotte Methuen (1998) identified four historical relationships between mathematics and religion: conflict, independence, dialogue, and integration. These ideas can be useful in discussing Charles' relationship to his religion and mathematics profession. In the previous paper about Joseph (Norton, 2002), I suggested that he seemed to hold an *independent* relationship between his mathematics and his Jewish religion. For Charles, I argue that the relationship is one of *conflict* followed by *integration*. Methuen identified the relationship for 16th century mathematician Philip Melanchthon as one of integration as well, though without the preceding conflict. That is, while Philip Melanchthon's philosophy that "the study of mathematics offers a vehicle by which the human mind may transcend its restrictions and reach God," (Methuen, 1998, p. 83) makes mathematician and pastor one, Charles doesn't see mathematics serving such a distinguished role.

Charles' struggle for mathematical meaning and value of mathematical practice began in childhood. When he prayed to God to save his parakeet and God responded, Charles was committed to keeping his promise of serving God in the best way that he could. Initially this promise stood in the way of his mathematical career. He knew very early in his life that he wanted to do research in mathematics and his teachers continually recognized his talent. But he felt that in order to fill his promise he might have to become a pastor because it was difficult for him to find religious meaning for his mathematical activity. However, he seemed to value doing things to help others, as a way of serving God. At first his view of efficacious service was restricted to direct human service, such as tackling environmental issues as an engineer. But later Charles found religious value in bringing Truth to the world, even among the secular sciences.

Charles believes that God orchestrates everything that happens in the Universe. This belief holds for both mathematical advancement and religious prophecy. In this way, Charles can serve God by helping to bring mathematical Truth to the world, so that "it's ok" for him to be a mathematician. However, the domain of mathematics does not stand out in importance from other secular studies, and the path toward Truth in these fields is not a direct one. In all domains of study, we are "wavering toward a truth." The ideas we hold today were brought to the world by God and through us, but they can still be proved false in the future. That is, by continually developing new ideas (with God's help), we are getting closer to Truth. In sum, Charles' view helped to integrate his mathematical practice and religious beliefs.

Like the twentieth-century mathematician Paul Erdös (Hoffman, 1998), Charles believes that there is absolute mathematical Truth. Erdös imagined a book in which all mathematical truths were written and jealously guarded by "the Supreme Fascist." Hardly a religious man, Erdös explained that "you don't have to believe in God, but you should believe in the Book" (p. 26). For Charles, on the other hand, the Book is held by God and the ideas that we are able to bring to the world may only be *leading toward* the Truth. Though Charles singles out mathematics as the most certain of the sciences, he does not feel that God's book is limited to this domain.

Like the Hindu mathematician, Ramanujan (Hoffman, 1998), Charles believes that God's method of dissemination is often very direct. Ramanujan claimed that his great mathematical ideas were delivered to him in his sleep, by the goddess Namagiri. Charles' connection to divine ideas is based, instead, on one of his central religious tenets: God answers prayer. The immediate relevancy of the ideas he receives in reply to prayer has convinced Charles that God often participates in Charles' mathematical activity in a very direct way. This belief is the strongest suggestion that Charles' mathematics and religion are integrated. Also in this way, he feels that his career has been blessed.

Finding Value in Mathematical Activity

In the paper about Joseph, I pointed out the meaningfulness of Joseph's "meritorious activity" as a mathematician. Joseph was raised with a religious value for secular study so that his mathematical pursuits were never in conflict with his religious beliefs. On the contrary, his mathematical pursuits were encouraged and possibly motivated by his religious beliefs. In fact, Joseph approached mathematical study in much the same way he approached his religious study of the Talmud. The case is very different for Charles who had to struggle for many years in search of mathematical meaning. His mathematical talents and interests remained at odds with his religious beliefs throughout most of his youth as he tried to reconcile the two domains.

While Christianity certainly does not preclude scientific and mathematical thought, we have seen how one particularly bright Christian mathematician struggled in coming to peace with his profession. The difficulty derives from the absence of value for secular studies in many Christian communities. Whereas this value was embedded in Joseph's Jewish religion, Charles had to undergo the arduous task of building it up on his own. His somewhat reclusive childhood may have aggravated the task. Perhaps if he could have engaged in dialogue with other Christian mathematicians about their perspectives, he might have been spared some of the anxiety. Herein lies the important message of Charles' story.

If students hold religious beliefs that do not value mathematical study, they are not likely to be motivated to overcome many of the cognitive struggles they experience in learning mathematics. As teachers in secular schools, we cannot foster a community for them to share religious perspectives and build religious meaning for mathematical study. However, we can strive to help them to find, in their Christian lives, a need for mathematics and a safe place—i.e., without religious conflict—in which to practice it.

Charles experienced conflict between scientific Truth and religious Truth very early in life, over the debate on creationism and evolutionism. Mathematics is safe in the sense that it need not make any claims about Truth at all, much less ones that might contradict religious Truth. Mathematics, in one sense, is a game played with logical rules and based on a few initial assumptions—none of which make any claims about the physical world or the nature of the spirit. In another sense, mathematics is a tool that can be applied in various fields that operate on additional assumptions in order to draw logical conclusions. If the conclusions within these other fields contradict one's religious beliefs, one can dismiss the assumptions of those fields. These perceptions of mathematics are not only safe, but more aligned with modern philosophy of mathematics than Melanchthon's perception of mathematics as "the vehicle to God" (Methuen, 1998, p. 83) or Erdös' lofty regard for "the Book" (Hoffman, 1998, p. 26). Morris Kline's *Loss of Certainty* (1980) provides ample evidence to demonstrate that mathematics is a human and fallible endeavor.

On the other hand, mathematics should still provoke a sense of amazement for its power to model and predict events and for the beauty of its inter-connectedness. Both of these aspects of mathematics allude to the need for it, but this perception of need may be circumvented if one perceives that religion offers a priori answers for all of life's needs. What need do students (Christian or otherwise) have for solving mathematical problems if everything we need to know can be found in a religious text or through divine intervention? Moreover, if mathematics is not an initially satisfying activity for students (unlike Charles), why should they seek its meaningfulness or necessity as Charles did?

In posing problems, mathematics educators should try to appeal to students' curiosity and sense of wonder. If mathematical problems appeal to students, as they did to Charles, we have a nice start. However, this appeal was not enough for Charles. He needed to know that his activity served a greater purpose. If mathematics is not "the vehicle to God" that Melanchthon imagined, maybe it is the application of mathematics in helping people to solve worldly problems that makes it a worthwhile and meritorious activity. Finally, as Charles concluded, it may be that we are all doing our part to bring God's truth to the world. While Biblical Truth will be most essential to many Christians, it is possible to attribute *all* knowledge to an omniscient God, and whatever parts people play in sharing that knowledge, it contributes to the whole. Charles' assumption that mathematics is the most certain of all sciences may explain why mathematics is so central to the development of knowledge and why mathematics serves a key role in so many of the parts we play.

REFERENCES

Glesne, C. (1999). *Becoming qualitative researchers: An introduction* (2nd ed.). New York: Addison Wesley Longman.

Hoffman, P. (1998). *The man who loved only numbers.* New York: Hyperion.

Kline, M. (1980). *Mathematics: The loss of certainty.* Oxford: Oxford University Press.

Methuen, C. (1998). *Kepler's tübingen: Stimulus to a theological mathematics.* Sydney, Australia: Ashgate.

Nord, W. A. (1999). Science, religion, and education. *Phi Delta Kappan, 1*, 28-33.

Norton, A. (2002). Mathematicians' religious affiliations and professional practices: The case of Joseph. *The Mathematics Educator, 12*(1), 17-23.

Singham, M. (2000). The science and religion wars. *Phi Delta Kappan, 2*, 425-432.

In Focus…

Inequity in Mathematics Education: Questions for Educators

Julian Weissglass

Many years ago I encountered a diagram (Figure 1) that may be familiar to you. It was used to help teachers understand that student learning depended upon the relationship between the teacher, the student, and mathematics. Although I found it helpful in thinking about my teaching, I eventually realized that there are many more triangles that affect student learning. One can draw triangles with students, teachers, parents, school board members, legislators, etc.

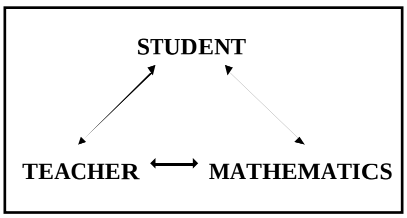


Figure 1. The relationship between the teacher, the student, and mathematics.

Figure 2 is another way of acknowledging that there are many factors that affect student learning and the well-publicized acheivement gap between students from different ethnic and socio-economic groups. The student/teacher/mathematics triangle is located in a classroom, in a school, in a district, in a community that is situated in a larger society. People in this community and in the larger society hold beliefs, attitudes, values, and often deep emotions about a variety of issues—teaching, learning, assessment, the nature of mathematics, the nature of schools in a democratic society, race, class, gender, sexual orientation, culture, and language—to name a few. In this article I will pose some questions and offer some thoughts about how some of these beliefs, attitudes, values, and emotions affect inequity in mathematics education.[1] The first question concerns mathematics and culture.

Julian Weissglass is Professor of Education at the University of California Santa Barbara. He received his Ph.D. in Mathematics from the University of Wisconsin, Madison and was a member of the UCSB Mathematics department for over 30 years. He is currently director of the National Coalition for Equity in Education and is most interested in developing leadership for educational equity.

Is mathematics culture free?

Some people say that mathematics is a set of eternal truths that humans discover. Others maintain that it develops from human social interaction—as all other forms of knowledge do. For example, Paul Ernest points out, “The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction” (1991, p. 42). It is not my intent to settle this dispute in this article. If mathematics is not culture free, however, then one might wonder:

Would mathematics be different if male European culture had not become the dominant force in the world?

Since mathematics develops in part to solve the problems of society, the culture of a society at least influences the course of mathematical development. Although this influence may seem far removed from the classroom, as soon as we start talking about mathematics problems, we are close to classroom issues. I think immediately of the distinguished mathematician George Polya (1887–1985), a strong advocate for developing a pedagogy of mathematical problem solving. He wrote “An essential ingredient of the problem is the desire, the will and the resolution to solve it. The problem that you are supposed to do and which you have quite well understood is not yet your problem. It becomes your problem, you really have it when you decide to do it.” (1962, p. 63) This comment leads me to another question:

How does a student’s culture, class, and gender affect whether the problem becomes her or his problem?

The first time I taught a class in mathematics for future elementary teachers, I chose a book that used counting problems to motivate the study of number systems. Many of the examples consisted of counting the number of different poker hands. These problems did not interest my students, 95% of whom were female. Mathematically the book was very sound, but it did not work for my students. The lesson for me was that just because *I* think a mathematical problem is fascinating does not mean that my students will find it so. If I do not enable my students to see mathematics

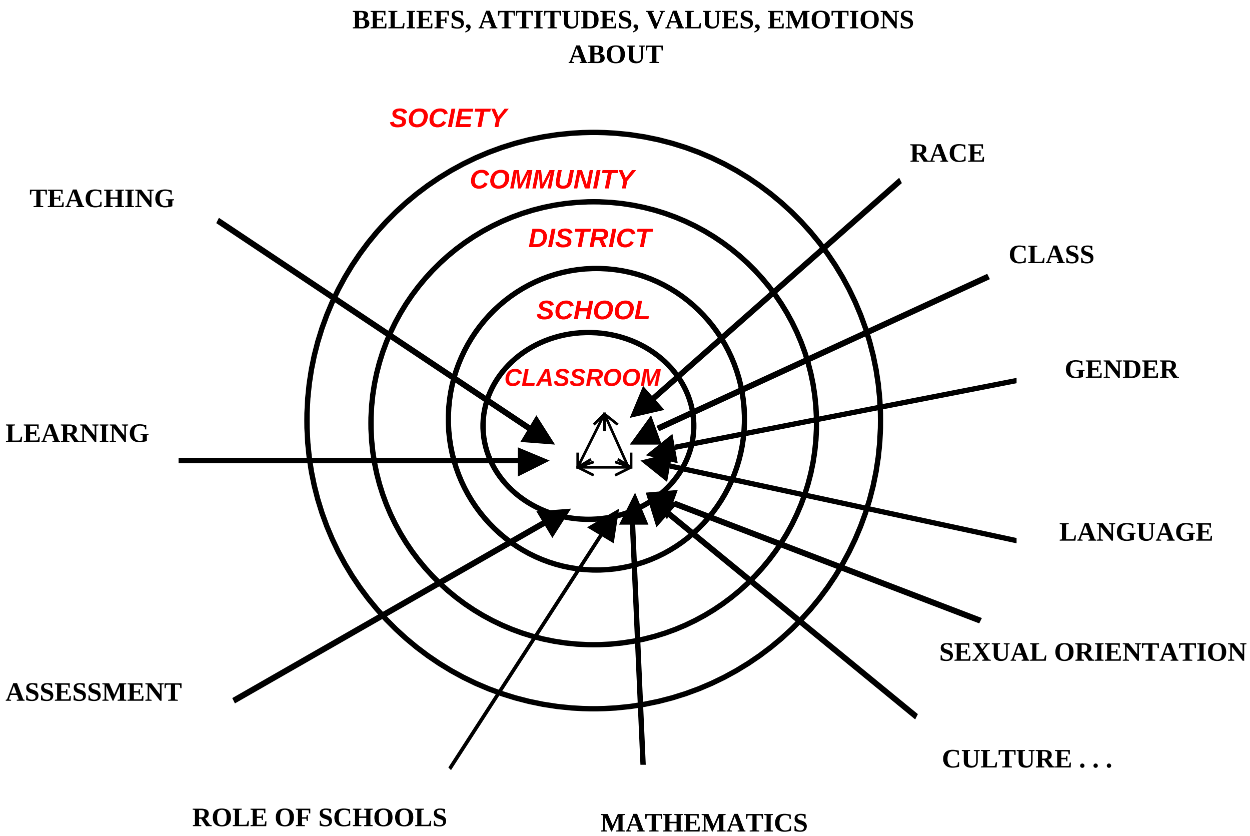


Figure 2. The many factors that affect student learning.

as useful to solve problems that are important to them, I lose a powerful motivational tool—and often lose lots of students. Since I do not want to lose students, I ask:

How do teachers and curriculum developers present problems that are likely to become the students' problems?

This question is closely related to culture, because the culture you grow up in affects what problems engage you. As Steig Mellin-Olsen put it, the probability that a topic "is recognized as important by a pupil is dependent on how he relates it to matters influencing his total life situation" (1987, p. 35). Mellin-Olsen gives three criteria for facilitating the development of this relationship: (See Weissglass, 1991, for a review of Mellin-Olsen's book.)

1. Mathematics is presented both with regard to the individual history of the student and the history of the culture from which the student comes.
2. Learning a skill occurs in the context of a wider project of interest to the learner.
3. Learning occurs within the context of cooperation. The gains of the individual feed the gains of the group.

Although young people are inherently curious about mathematics when they enter school, most do not maintain this curiosity. So we can ask:

How do educators' culture, class, and gender affect their ability to develop and communicate problems so that students desire to solve them?

Herein lies a major challenge. Growing up in U.S. society often causes white middle class children to be misinformed or ignorant about history and the lives of people with different backgrounds than themselves. Since most curriculum developers come from this pool, the concerns of racial and ethnic minorities are often ignored in curriculum. Even the most progressive curricula usually do not relate mathematics to the concerns of students who are living in poverty. Curriculum developers often choose problems that are "safe" and uncontroversial, routinely neglecting issues related to social justice. As Kastberg pointed out in a previous issue of this journal, the writers of the NCTM *Principles and Standards for School Mathematics* (2000) "ignore the power that mathematics can have in debate and discussion of issues critical to the elimination of social and economic inequities in the United States" (Kastberg, 2001, p. 18).

Sometimes students are told blatant falsehoods. For example, attempting to situate an algebra lesson in a historical situation, the authors of a highly-regarded eighth grade mathematics text wrote, "When Mexico ceded California to the United States in 1848, California was a relatively unexplored territory with only a few thousand people" (Lappan, Fey, Fitzgerald, Friel, & Philips, 1998, p. 5). In fact anthropologists estimate that there were approximately 150,000 indigenous people in California at that time. Furthermore, to say that "Mexico ceded California to the United States" without mentioning that the U.S. military was encamped outside Mexico City ready to conquer the whole country is misleading. It is akin to saying that in the 17th century large numbers of Africans came to North America to help grow cotton, without mentioning slavery.

Because I suspect that curriculur materials might change if the community of curriculum developers were more diverse, we can also ask the following:

How do we increase the percentage of people of color in curriculum development groups?

Responding to this question is beyond the scope of this article, but I conjecture that it will require an effort similar to the one I recommend later for schools.

If you accept that culture, class, and gender affect teachers and students in the math classroom, then you are ready for the next question:

How do racism and classism affect the school experiences of students?

Since racism and classism are words that have different meanings for people, let me define what I mean by them.[2]

> *Racism* is the systematic mistreatment of certain groups of people (often referred to as people of color) on the basis of skin color or other (real or supposed) physical characteristics. This mistreatment is carried out by societal institutions or by people who have been conditioned by the society to act, consciously or unconsciously, in harmful ways toward people of color.
>
> Similarly, *classism* is systematic mistreatment based on socioeconomic status. This mistreatment includes prejudice, discrimination, disrespect, or neglect of rational needs (food, clothing, shelter, communication, opportunities to learn, etc.)

The racism and classism pervasive in U.S. society influence the attitudes that people develop. Teachers carry these attitudes with them into their classroom. For example, they may have different expectations for individuals from different groups. But the situation is more complex than that. I cannot, in this article, fully explain how racism and classism and other forms of systematic mistreatment work in schools, but let me at least point out that they can be blatant or subtle, personal or institutionalized, conscious or unconscious.

The perception of whether an act is blatant or subtle will vary from person to person. If a teacher or a textbook author limits themselves to problems or situations that avoid social justice issues, someone might say that that is subtle classism. On the other hand, a person who is suffering the effects of classism might see that action as a blatant. Similarly, a policy at the institutional level can contribute to the systematic mistreatment of a group even though no one consciously attempted to hurt them. For example, California implemented a class size reduction policy a few years ago. It caused an immediate teacher shortage and many experienced teachers from inner cities in California took jobs in the suburbs, probably because they thought it was easier to teach there. The result was that students in inner city schools were being taught by less experienced teachers. The effects on poor students and students of color were severe—an example of unconscious institutionalized racism/classism. There is a general principle at work here. Whenever policy deliberations do not include an informed consideration of how racism/classism operate in society, the policy developed will probably not be for the long-range benefit of people of color and people in poverty and will often work to their disadvantage.

There are many examples of the blatant racism in the history of the United States—slavery, lynching, segregation, discrimination in housing. It was perfectly respectable for academics (including eminent scientists) in the 1920s and 1930s to join the eugenics movement, which was based on an ideology of racial superiority (See Gould, 1981; Tucker, 1994). For example, Edward East, a Harvard geneticist, wrote: "Gene packets of African origin are not valuable supplements to the gene packets of European origin; it is the white germ plasm that counts" (1927, p. 199). But these pseudoscientific ideas have not disappeared. In a well-publicized book in 1994, the authors wrote "Putting it all together, success and failure in the American economy and all that goes with it, are increasingly a matter of the genes that people inherit" (Herrnstein & Murray, p. 91). While discussing this book a mathematics professor told me, "Anyone who is intelligent has made his way out of the working class."

We like to think things have gotten better, and I believe that in many ways they have—but I am white and middle class. Two examples from California, separated by almost 70 years, suggest that perhaps racist practices and policies have just become subtler. In a 1923 address to the Los Angeles Unified School District principals the Superintendent, Susan B.

Dorsey, said “We have these Mexican immigrants to live with and if we can Americanize them, we can live with them.” (García, 2001, p. 51). In 1991, a superintendent of a school district in northern California said, “We’ve got to attend to the idea of assimilation and to make sure that we teach English and our values as quickly as we can so these kids can get in the mainstream of American of life” (p. 51). Many people have a different perspective on culture. César Chávez, for example, said, “Preservation of one’s own culture does not require contempt or disrespect for other cultures” (Chicano Studies Research Library, 1996).

Racist/classist practices do not have to be overt. Ignoring issues of race and class are widespread. For example, at the National Summit on the Mathematical Education of Teachers (co-sponsored by the National Science Foundation [NSF], the Exxon Mobil Foundation, and the Conference Board of the Mathematical Sciences) in November 2001, only one of the titles and abstracts made even a passing reference to the challenges of teaching “algebra for all”. None mentioned the challenges of teaching mathematics to a culturally diverse student population or to language minority students. None addressed how teachers’ expectations and belief systems affect the teaching and learning of mathematics, or any of the other issues that are related to the achievement of under-represented students in mathematics. Were these omissions subtle or blatant, conscious or unconscious? From my point of view, the silence was deafening.

Evaluation (assessment) methods are another area where racist/classist practices may not be overt. I wonder:

How much of the assessment system is driven by (unconscious) race and class bias?

Ubi D’Ambrosio, the Brazilian mathematics educator, points out, “… mathematics has been used as a barrier to social access, reinforcing the power structure which prevails in the societies (of the Third World). No other subject in school serves so well this purpose of reinforcement of power structure as does mathematics. And the main tool for this negative aspect of mathematics education is evaluation.” (1985, p. 363) Many mathematics educators find this statement puzzling. How can evaluation (assessment) be used as a barrier to social access? Consider the following:

1. The nature of the instrument may incorporate cultural values and practices—such as being able to respond quickly on timed tests or being good at figuring out how to eliminate answers in multiple choice questions.
2. Previous experiences with being asked questions and riddles and being rewarded for the right response, might be valued in upper-class and middle-class homes more than it is in working class or poverty-stricken homes.
3. The test-taking environment might work to the disadvantage of students from different cultural and class groups by causing different levels of anxiety.

In regard to the third point, social psychologists have shown that the performance of members of nearly any stereotyped group can be negatively affected by manipulating (sometimes very subtly) the conditions of the testing environment (through instructions or questions given to the test-takers) to bring to consciousness or sub-consciousness one’s membership in that group. For example, Steele and Aronson (1995) found that African American college students performed significantly worse than Whites on a standardized test when the test was presented as a diagnosis of their intellectual abilities, but about as well as Whites when the same test was presented as a non-evaluative problem-solving test. Other researchers (Croizet & Claire, 1998; Shih, Pittinsky, & Ambady, 1999) produced similar results with different groups. Social psychologists use the term *stereotype threat* to describe this phenomenon. I prefer the term *internalized oppression*—the phenomenon of people believing in the messages they receive from society and, as a result, acting in hurtful ways to themselves. A Latino principal talking on a Personal Experience Panel at an institute conducted by the Equity and Mathematics Education Leadership Institute (EMELI, n.d.) sheds some light on the process:

> …it happened slowly and you know what’s going on but you can’t understand it. …like the SRA, the reading classes. …there’s different colors [for different levels]. I was always in the lower one. I was treated a little bit different again because I was in this lower group and I started noticing a lot of my buddies were in the same group I was in and a lot of the other kids that were usually quiet were in the higher groups and you start kind of feeling a little bit less. You start feeling less about yourself. …as I went into high school, they have the tracks A, B, and C. And C is just one step above Special Ed. And again, I was in the C group and my buddies were in the C group with me. You know…people treat you differently. As I got into college I always felt inadequate, not being capable to do these things.

Experiencing racism (or other forms of mistreatment) outside of school can also affect school performance. A Japanese American teacher speaking on a Personal Experience Panel recounted a memory when the parents of a white girl he met did not allow her to date him. He continued:

> That's pretty much the age when my whole attitude toward achievement changed. Before then I had been very competitive…striving to be the top student by taking at least eight classes a day and going to special GATE type programs on weekends and summers. I was president of many extracurricular clubs. I ran for student body offices, played in the jazz band, did athletics, and gave speeches in every oratorical contest I could find. A year later, my final schedule consisted of coming to school at 11 AM for two classes and basketball practice. I went out with the boys every evening and weekend. …I was very lucky not to be expelled for drinking (as my two buddies were). (See Weissglass, 1997, for the more complete story.)

So I ask:

Can we change racist/classist practices in schools and eliminate (or at least alleviate) the effects of racism and classism on students?

I think we can, but good intentions, hard work, excellent curriculum, or research, will not be sufficient. Significant change in people's beliefs, attitudes, and practices only comes through a complex process of sense-making, reflection, and re-evaluation of existing practices and understandings. This process of re-evaluation is facilitated by being listened to and releasing emotions about the experiences that formed our beliefs, attitudes, and behavior patterns.

Shirley Chisholm points out, "Racism is so universal in this country, so widespread and deep-seated, that it is invisible because it is so normal" (1970, p. 133). Therefore, we need school communities where it is safe enough for the invisible to be made visible—where Whites can listen to people of color talk about how they and their ancestors have experienced racism and where people of color can listen to Whites talk about how seeing racial prejudice has affected them. Listening to each other's stories and emotions helps people identify what needs to change within their institutions and themselves. Being listened to will help them heal from the hurts of racism. Professional therapists are not necessary, nor are there enough of them to do the job. It is one of our responsibilities as educators to heal ourselves from the damage racism has done. However, there is a tendency among educators to dismiss healing from hurt as too "touchy-feely" to belong in academic institutions.[3] But consider that this country has spent hundreds of millions (perhaps billions) of dollars in the last two decades on attempts to decrease the differential success in mathematics between students from different ethnic groups without any major change on the national level. It is clearly time to risk new approaches.

The National Coalition for Equity in Education (NCEE, n.d.) has developed a theory and a set of structures and approaches that help educators productively address racism, classism, and other forms of bias.[4] Three (of the twelve) assumptions underlying the work of NCEE are:

- No one is born prejudiced.
- All people are deeply hurt by growing up in a racist and classist society.
- Many of the assumptions, values and practices of people and institutions hinder the learning of students of color and students from low-social economic classes.

Let me emphasize the first assumption. I believe that human beings are good. No one would hurt another person or participate passively in a hurtful system unless they had been hurt. Nevertheless, unequal access to resources, violence and threats of violence, miseducation, lies, stereotypes, and disrespect (some of which are carried out, and others accepted, by educational institutions) cause great harm to people of color and people living in poverty. If we avoid facing this we are silent bystanders—contributing by our silence to the perpetuation of racist and classist practices.

I propose that we adopt a new paradigm for our schools: *schools as healing communities*. People in such schools will undertake a wide range of anti-bias work. Educators will identify how their unconscious bias is affecting their students and challenge any low expectations they or their colleagues hold. Parents and teachers will work together to support children's learning. Members of the community will identify how bias is institutionalized in policies and practices and they will work for change. Teachers will question their curriculum and pedagogy and make it more engaging to students of different cultures. Schools will teach the history of how oppressed peoples have been treated in this country. They will support students and their families to challenge internalized oppression. Students will talk about and heal from how they experience unfairness and discrimination. A healing community will have as its highest priority people caring about students and their learning. True learning, as contrasted with rote memorization for rapid recall, will increase.

Achieving such communities will require leadership—a different kind of leadership than in other reform areas. The issues related to racism, classism, and other forms of bias are complex and emotional. Some people, for example, will not recognize racism or classism. Some may deny that they interfere with their relationships ("I don't see color"; "I treat everyone the same") or affect institutional or governmental policies.

People of color may feel hopeless or cynical about the possibility of change. They may be skeptical of Whites making a commitment to combat racism. Students and teachers may be fearful of talking honestly about racial or class issues. Leaders will need to understand the personal, social, and institutional roots of inequities. They will be able to raise controversial issues while building unity, relate well with people from different backgrounds, help people recover from hopelessness and powerlessness, and deal constructively with their own and others' emotions. These skills and knowledge are not routinely developed in schools or colleges or in professional development.

Creating healing communities will require resources. Not allocating these resources will be even more costly in the long term. It is easier for educators to hold a workshop celebrating diversity, develop curricula, buy "test-prep" programs, pressure teachers, or write reports and vision statements than to talk about personal experiences with race and class. But any reform effort attempting to reduce the achievement gap that does not get at the roots of racism/classism will not be likely to succeed over time. If, as a nation, we develop healing communities where people can speak honestly about racism/classism and heal from their hurts, we can change biased practices, attitudes, and policies. If we can communicate caring to students, and help them recover from racism/classism, they will be able to achieve their full human potential. If we do all this, we will accomplish more than reducing the achievement gap in mathematics. We will create a better society.

REFERENCES

Chicano Studies Research Library. (1996, June). *Sï Se Puede! Cesar E. Chavez and His Legacy.* Retrieved from http://clnet.ucr.edu/research/chavez/quotes/cult.htm

Chisholm, S. (1970). *Unbought and unbossed.* Boston: Houghton Mifflin.

Croizet, J. C., & Claire, T. (1998). Extending the concept of stereotype threat to social class: The intellectual underperformance of students from low socioeconomic backgrounds. *Personality and Social Psychology Bulletin, 24*, 588-594.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

East, E. M. (1927). *Heredity and human affairs*. New York: Scribners.

Equity and Mathematics Education Leadership Institute [EMELI]. (n.d.). *What is EMELI?* Retrieved October 20, 2002, from http://emeli.education.ucsb.edu

Ernest, P. (1991). *The philosophy of mathematics education*. Bristol: The Falmer Press.

García, E. E. (2001). *Hispanic education in the United States: Raíces y alas*. Lanham: Rowman & Littlefield.

Gould, S. J. (1981). *The mismeasure of man*. New York: W.W. Norton.

Herrnstein, R. J., & Murray, C. (1994). *The bell curve: Intelligence and class structure in American life*. New York: Free Press.

Inverness Research Associates [IRA]. (2001, February). *Educators for equity: The work of EMELI leaders*. Retrieved from http://www.inverness-research.org/reports/ab_emeli.html

Kastberg, S. (2001). Problem context in the Standards: What is the message? *The Mathematics Educator, 11*(1), 15-19.

Lappan, G., Fey, J., Fitzgerald, W., Friel, S., & Philips, E. (1998). *Frogs, fleas, and painted cubes: Quadratic relationships*. Menlo Park: Dale Seymour Publications.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: D. Reidel.

National Coalition for Equity in Education [NCEE]. (n.d.). *Mission*. Retrieved October 20, 2002 from http://ncee.education.ucsb.edu

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Polya, G. (1962). *Mathematical discovery, Vol. 2*. New York: John Wiley & Sons.

Shih, M., Pittinsky, T., & Ambady, N. (1999). Stereotype susceptibility: Identity salience and shifts in quantitative performance. *Psychological Science, 10*(1), 80-83.

Steele, C. M., & Aronson, J. (1995). Stereotype threat and the intellectual test performance of African Americans. *Journal of Personality and Social Psychology, 69*(5), 797-811.

Tucker, W. H. (1994). *The science and politics of racial research*. Urbana, IL: University of Illinois Press.

Weissglass, J. (1990). Constructivist listening for empowerment and change. *The Educational Forum, 54*(4), 351-370.

Weissglass, J. (1991). Reaching students who reject school: An essay review of *The Politics of Mathematics Education*. *Journal of Mathematical Behavior, 10*(3), 279-297.

Weissglass, J. (1997). *Ripples of hope: Building relationships for educational change*. Santa Barbara, CA: Center for Educational Change in Mathematics and Science.

Weissglass, J. (2000). No compromise on equity in mathematics education: Developing an infrastructure. In W. Secada (Ed.), *Changing the faces of mathematics* (pp. 67-78). Reston, VA: NCTM.

[1] If you are reading this article with a group of people, or you have someone nearby, I suggest that you pair up and each spend a few minutes talking about the beliefs, attitudes, values, and emotions about the topics in Figure 2 that most affect you in your work.

[2] If you are reading this article with a group of people, I suggest that you pair up and give each person get a chance to talk for two or three minutes about how she or he feels when she or he hears the word racism.

[3] I discuss why the culture of schools does not respect emotional release in Weissglass (1990).

[4] The theory and approaches, as well as examples of stories told by educators on Personal Experience Panels, are described in Weissglass (1997). The way the theory and structures is used in equity work is described in Weissglass (2000). The evaluation of a project based on this approach (IRA, 2001) is available on the web.

Book Review…

A Critical Discourse Offered: A Review of *Radical Equations: Math Literacy and Civil Rights*

David W. Stinson

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights*. Boston: Beacon Press.
ISBN 0-8070-3126-7

In 1989 the National Council of Teachers of Mathematics (NCTM) listed “Opportunity for all” as one of the “New Societal Goals” for mathematics education (pp. 3-4). By this time Robert P. Moses had already been taking that idea to communities in the form of the Algebra Project, a grassroots education movement he originated and developed to increase black students’ access to algebra as a gateway to the study of advanced mathematics. A historical and active civil rights leader, Moses is an expert at bringing novel ideas to communities even if the ideas seem impossible to implement. He equates rallying support and action from the Mississippi sharecropper for the “one person, one vote” slogan of the Mississippi Summer Project[1] to rallying support and action from students, parents, and school and community leaders for the “If *we* can teach students algebra in the middle school years, then *we* should do it” (pp. 91-93) slogan of the Algebra Project.[2] His justification for this “radical” equation and description of how the lessons learned from Mississippi assisted in the development of the Algebra Project are the central topics of his book *Radical Equations: Math Literacy and Civil Rights* (2001), co-written with journalist Charles E. Cobb Jr.

The purpose of Moses’ personal and compassionate narrative is not to provide tenets of a reform curriculum and research results of its effectiveness for scholarly critique, but to get readers to rally behind the idea that to deny a student access to an advanced mathematics education is in essence to deny them their civil rights, and that to achieve mathematical literacy for all students will take grassroots activism. Thus the intended audience of the book is broader than the readers of academic journals: Moses wants to reach not only mathematics educators with his message, but students, parents, and leaders of schools and communities. In this review, I argue that Moses’ book is an important read for all those involved in mathematics education because of the critical discourse he provides on two key aspects of reform. In addition, I discuss boundaries of Moses’ approach and limitations that stem from Moses’ lack of questioning in three areas that may be critical to reform efforts.

David W. Stinson is a third-year doctoral student in mathematics education at the University of Georgia. His research interests are the socio-cultural aspects of mathematics and mathematics teaching and learning.

Through interweaving his personal experience as a civil rights leader, Moses engages the reader with *conscientização*[3] in two recurring aspects of reform in mathematics education. The first aspect, the need for reform, is often located in discussions regarding mathematical literacy requirements of the future workforce of the U.S. economy. Arguing the need for reform within this frame of reference is not new; “Mathematically literate workers” (NCTM, 1989, p. 3) was one of the “New Societal Goals” stated in the *Standards*. And like the NCTM and other educational and government organizations that have discussed the issue before and after the release of the *Standards*, Moses presents the need for reform by providing statistical facts about the new high-tech information-age service economy of the United States in which mathematically literate workers are essential.[4] Furthermore, he identifies an advanced mathematics education as the key to becoming technologically literate in this economy.

Moses provides a critical discourse on this need for reform by not only acknowledging mathematical literacy for all as an “economic necessity” (NCTM, 1989, p. 4), but by communicating that mathematical literacy for all is necessary for the attainment of civil rights. Moses believes history has demonstrated that being denied economic and educational access leads to being denied full citizenship. That is, by examining the U.S. economic and education structures from the post-Civil War period to the twentieth-century, he concludes that poor people and people of color continue to be subjugated to positions of economic disadvantage and unjust schooling practices. As a result, these people are effectively prevented from attaining full citizenship. He equates the oppressive elements of the institution of sharecropping with the unjust schooling practices of the institution of public schools that often provide poor students and students of color with a “sharecroppers education” while they prepare “an elite to run society” (p. 11).

Moses identifies a specific component of the sharecropper education as the failure to prepare students for advanced mathematical studies in their middle school years, which results in students' lack of opportunity to participate in an advanced mathematics curriculum in their high school years. Moses argues this lack of opportunity most often means students will not develop mathematical literacy. Moses believes mathematical illiteracy will result in limited economic access, and he sees participation in algebra as crucial.[5] Although Moses admits that there has always been a certain level of cultural acceptance for mathematical illiteracy in the United States, he claims it affects Blacks and other minorities more severely—"making them the designated serfs of the information age just as the people that we [civil rights activists] worked with in the 1960s on the plantations were Mississippi's serfs then" (p. 11).

Moses also creates critical discourse through detail. By explicitly identifying white power structures as oppressors, Blacks and other minorities as being oppressed, and the operation of oppression in denying them economic access and full citizenship, Moses makes explicit the NCTM's statement that "[t]he social injustices of past schooling practices can no longer be tolerated …[m]athematics has become a critical filter for employment and full participation in our society" (1989, p. 4). That is, Moses specifies injustices in ways that the NCTM cannot (or dares not). Thus he achieves a certain critical power: In light of his words, it becomes more difficult for educators to ignore or brush off inequities such as the continued imbalances in the race of students in advanced mathematics classrooms in the U.S.

Enacting reform in mathematics education is the second aspect of reform for which Moses provides a critical perspective. Based on his experience of developing grassroots organizations that took action against the oppressive U.S. political structures during the 1960s Civil Rights Movement, Moses believes that it will take similar grassroots organization and action in order for Blacks, and other minorities, to implement mathematics education reform within their communities. This belief is also based on his observations of urban and rural school communities that currently implement the reform curriculum of the Algebra Project in their efforts to provide students equal access to an advanced mathematics education. Moses attributes the successful growth of the Algebra Project to the two main tenets borrowed from his experience in the Mississippi Summer Project: "the centrality of families to the work of organizing, and organizing in the context of the community in which one lives and works" (p. 18). He believes that the most important lesson learned from Mississippi is that "it is getting people at the bottom to make demands, on themselves first, and then on the system, that leads to some of the most important changes" (p. 20). In other words, only after students, followed by parents, and then by school and community leaders, organize and take action against the oppressive elements of schools and schools systems will all children be provided the opportunity to participate in an advanced mathematics education.

To illustrate the power of grassroots organization when implementing reform, in Part I of his book, Moses recounts the story of his involvement in the Civil Rights Movement. This story engages the reader in Moses' personal encounters with such figures as Ella Baker, Martin Luther King Jr., John Lewis, Julian Bond, Medgar Evers, and others. But more importantly, in this section Moses relates his encounters with "ordinary working people", black and white, who through organizing became self-empowered and facilitated change. Moses says that it was these individuals who not only took the lead in challenging white power but also challenged and changed themselves.

Part II of Moses' book chronicles the actual enactment of his mathematics education reform efforts, the development of the Algebra Project. The Project began in 1982 when Moses taught Algebra I to his daughter and three of her classmates because the course was not offered at their Cambridge, Massachusetts public middle school. This initial instruction developed into community and school meetings, initially organized by Moses and then by community members, addressing the need and desire of the students to have the opportunity to enroll in Algebra I at their middle school. Over the past twenty years these meeting have developed into a network of Algebra Project Programs in over twenty cities, serving more than 40,000 children from urban and rural communities throughout the United States. Moses defines the driving force of the Algebra Project as the idea that the ongoing struggle for citizenship and equality is linked to the issue of math and science literacy; he describes the project as "a community organizing project—rather than a traditional program of school reform" (p. 18).

Moses states that the fundamental purpose of the Algebra Project curriculum is to prepare students for an advanced mathematics curriculum in their high school years. The curriculum[6] outlines five steps in which students participate when presented with a new mathematical idea. The new idea begins with a representative *Physical Event* that is shared by the class. This event is then followed by an *Individual Pictorial Representation/Modeling* and engagement in individual and group *Intuitive Language* about the physical event. This classroom discourse concludes with the introduction to *Structured Language* and then *Symbolic Representation* to describe the mathematical properties of the event. Throughout these steps students make sense of their own individual mathematical understanding as well as the cumulative mathematical understanding of the class. Moses describes the Algebra Project mathematics classroom as one that encourages interaction, cooperation, and group communities.

Near the end of the book, Moses links the activism of the 1960s to a new activism for young people—an activism that will ensure that all children are provided with the opportunity to enroll in an advanced mathematics education. Because the organizing and demands of the young people were instrumental in creating large-scale change in the 1960s, Moses believes "Movement emerges from Movement" (p. 193). A result of this belief is the emergence of the Young People's Project (YPP) out of the Algebra Project. The YPP allows students who have benefited from the Algebra Project to contribute by giving back to their community. The task of the YPP is to convince other young people and community members that mathematical literacy "is something they need, that acquiring it is work they should commit to, and that spreading it wherever they can is also their challenge" (p. 182). Moses believes that duplicating the reform strategy of the Civil Rights Movement when enacting mathematics education reform is not a "romantic longing for the past" (p. 17), but is the use of a proven strategy.

It is important for academic readers to understand that Moses' book has intentional boundaries in keeping with its purpose. For example, Moses limits the discussion regarding foundational details of the Algebra Project curriculum. However, he does provide readers with hints of the philosophical and pedagogical ideas that have motivated the curriculum. Moses refers to experiential learning traditions when he writes, "In other words, in the Algebra Project we are using a version of experiential learning; it starts with where the children are, experiences that they share" (p. 119). He also alludes to constructivism when he states that the "'social construction of mathematics' [is when s]tudents learn that math is the creation of people—people working together and depending on one another" (p. 120). And in discussing teacher practices he implicitly critiques a "banking" system of teaching and learning when he suggests that teachers "cannot simply be lecturers attempting to pour knowledge into the heads of students who sit passively like inanimate vessels" (p. 122). Although an informed reader can make inferences about the philosophy upon which Moses draws, formal references to education scholars and research are not included in the book.

Another boundary of Moses' book is the limited discussion of empirical evidence that may support the effectiveness of the Algebra Project. Rather than focusing on methodological details and results of research studies, Moses emphasizes the personal stories of students and communities that illustrate the positive effects of reform achieved through the Algebra Project. However, he does summarize a three-year empirical study that involved students from three Alabama elementary schools. Two schools were used as the control group, receiving "traditional" instruction, and one school was used as the experimental group, receiving instruction through the Algebra Project curriculum. At the end of the third year the researchers found that students in the experimental group were out-performing students from both of the control group classrooms on standardized tests.

While these boundaries are arguably necessary for Moses to achieve his purpose, I believe Moses' book also has limitations—issues that he does not deal with sufficiently and that relate closely to his purpose. In particular, I have identified three questions Moses does not, and perhaps should, address. First, although Moses questions to whom and how mathematics is taught in schools he fails to question the *mathematics* that is taught in schools. Moses discusses mathematics as though the discipline has been inscribed in some "great book"—as a static body of knowledge students must come to know. Does this orientation to mathematics imply that he believes the discipline only needs to be effectively transmitted in some way (though not by lecturing) to all students? In the past few decades, many mathematics educators have anchored their discussions about the discipline of mathematics in constructivism, which honors the individual mathematical knowing of the student. Although the Algebra Project curriculum encourages students' engagement and discourse in mathematics, it seems that the ultimate goal of the curriculum is to have all students conversant in a static mathematics that resides external to students in school textbooks.

Second, Moses does not question the role that mathematics plays in the schools as a "critical filter" for higher education. Moses argues that all students should have access to a college preparatory curriculum without arguing why an advanced mathematics education is a prerequisite for college. Does the absence of such an argument imply that he is content in positioning mathematics as a filter? Since Gardner's (1983) work in multiple intelligences, many in education have questioned the over reliance of verbal and mathematical skills in accessing "intelligence." Even though many in education would agree that mathematical literacy is necessary for an individual in the U.S. in the 21st century, many educators would also disagree that mathematical literacy should continue to be used as a filter for access to higher education.

Third, Moses does not question the structure of schools. Although he identifies past and current unjust schooling practices that are enacted on poor students and students of color, he does so without critically questioning the structure that enacts these practices. Does this lack of questioning imply that Moses believes schools to be just and democratic structures for middle class white suburban students? Some educators may argue that the unjust and undemocratic practices enacted on some students cannot be corrected within structures that enact questionable democratic schooling practices for all students. In fairness to Moses, I understand that to critically question schools as institutions is often unpopular and may be at cross-purposes to engaging in conversation with leaders of these institutions, one of Moses' central goals. However, I believe students, parents, school and community leaders, and the mathematics education community need to engage regularly in critical conversation that disrupts aspects of how these institutions function in U.S. society.

Despite these limitations, the book is an important read: The value of Moses' critical discourse on reform outweighs the lack of questioning in these areas. Indeed, the limitations may provoke *more* discussion about issues of equity and reform in mathematics education, and thus prove to be assets. In the end, Moses' experiences as a civil rights leader demonstrated to him, and the country, that effective transformation can occur even within a flawed structured if "ordinary working people" organize and take action. Along with Moses I hope that the same can be said of mathematics education in our schools.

REFERENCES

Freire, P. (2000/1970). *Pedagogy of the oppressed, 30th anniversary edition*. New York: Continuum.

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences.* New York: Basic Books.

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights.* Boston: Beacon Press.

National Council of Teachers of Mathematics [NCTM]. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

[1]The objective of the Mississippi Summer Project was the voter registration of African-American citizens in the State of Mississippi during the late 1950s and early 1960s.

[2] Moses defines *we* as a "complex configurations of individuals; educational institutions of various kinds; local, regional, and national associations and organizations (both governmental and nongovernmental); actual state governments as well as the national political parties, and the executive, legislative, and judicial branches of the national government" (p. 92).

[3] In *Pedagogy of the Oppressed* Paulo Freire (2000/1970) defined *conscientização* as "learning to perceive social, political and economic contradictions, and to take action against the oppressive elements of reality" (p. 35).

[4] For example, the Department of Labor has announced that by the year 2010 all jobs will require significant technical skill.

[5] Moses believes that the U.S. mathematical community has designated algebra as the starting point for an advanced mathematics education.

[6]The book's appendix provides a detailed example from the Algebra Project curriculum on the addition and subtraction of integers.

The Mathematics Educator (ISSN 1062-9017) is a biannual publication of the Mathematics Education Student Association (MESA) at The University of Georgia. The purpose of the journal is to promote the interchange of ideas among students, faculty, and alumni of The University of Georgia, as well as the broader mathematics education community. *The Mathematics Educator* presents a variety of viewpoints within a broad spectrum of issues related to mathematics education. *The Mathematics Educator* is abstracted in *Zentralblatt für Didaktik der Mathematik* (International Reviews on Mathematical Education).

The Mathematics Educator encourages the submission of a variety of types of manuscripts from students and other professionals in mathematics education including:

- reports of research (including experiments, case studies, surveys, philosophical studies, and historical studies), curriculum projects, or classroom experiences;
- commentaries on issues pertaining to research, classroom experiences, or public policies in mathematics education;
- literature reviews;
- theoretical analyses;
- critiques of general articles, research reports, books, or software;
- mathematical problems;
- translations of articles previously published in other languages;
- abstracts of or entire articles that have been published in journals or proceedings that may not be easily available.

The Mathematics Educator strives to provide a forum for a developing collaboration of mathematics educators at varying levels of professional experience throughout the field. The work presented should be well conceptualized; should be theoretically grounded; and should promote the interchange of stimulating, exploratory, and innovative ideas among learners, teachers, and researchers.

Submission by Mail:

Submit five copies of each manuscript. Manuscripts should be typed and double-spaced, conform to the style specified in the *Publication Manual of the American Psychological Association, 5th Edition*, and not exceed 25 pages, including references and endnotes. Pictures, tables, and figures should be camera ready. The author's name and affiliation should appear only on a separate title page to ensure anonymity during the reviewing process. If the manuscript is based on dissertation research, a funded project, or a paper presented at a professional meeting, a note on the title page should provide the relevant facts. Send manuscripts to the postal address below.

Electronic Submission:

Submit an attachment of your manuscript saved as a Microsoft Word or Rich Text Format document. The manuscript should be double-spaced and written in a 12 point font. It must conform to the style specified in the *Publication Manual of the American Psychological Association, 5th Edition,* and not exceed 25 pages, including references and footnotes. Pictures, tables, and figures should be in a format compatible with Word 95 or later. The author's name and affiliation should appear only on the e-mail message used to send the file to ensure anonymity during the reviewing process. If the manuscript is based on dissertation research, a funded project, or a paper presented at a professional meeting, the e-mail message should provide the relevant facts. Send manuscripts to the electronic address given below.

To Become a Reviewer:

Contact the Editor at the postal or email address below. Please indicate if you have special interests in reviewing articles that address certain topics such as curriculum change, student learning, teacher education, or technology.

Postal Address:
The Mathematics Educator
105 Aderhold Hall
The University of Georgia
Athens, GA 30602-7124

Electronic address:
tme@coe.uga.edu

The Mathematics Education Student Association is an official affiliate of the National Council of Teachers of Mathematics (NCTM). MESA is an integral part of The University of Georgia's mathematics education community and is dedicated to serving all students. Membership is open to all UGA students, as well as other members of the mathematics education community.

Visit MESA online at http://www.ugamesa.org

TME Subscriptions

A subscription is now required in order to receive the printed version of *The Mathematics Educator*. Subscribe now for Volume 13 (Numbers 1 & 2, published in the spring and fall of 2003).

To subscribe, send the information requested below along with the subscription fee to:

The Mathematics Educator
105 Aderhold Hall
The University of Georgia
Athens, GA 30602-7124

If you would like to be notified by email when a new issue is available online, please send a message to
tme@coe.uga.edu

- -

I want to subscribe to *The Mathematics Educator* for Volume 13 (Numbers 1 & 2).

Name ______________________________________

Address ____________________________________

____

____

Amount Enclosed ___________
($6/individual; $10/institutional)

In this Issue,