The Mathematics Educator
2010, Vol. 20, No. 1, 35-42

Mathematics and Martial Arts as Connected Art Forms

Serkan Hekimoglu

Parallels between martial arts and mathematics are explored. Misguided public perception of both disciplines, students' misconceptions, and the similarities between proofs and katas are among the striking commonalities between martial arts and mathematics. The author also reflects on what he has learned in his martial arts training, and how this wisdom influences his mathematics teaching. As a result of his martial arts training, his awareness of how he teaches mathematics has shifted, and his understanding of his students' struggles has deepened. Finding the balance between theory and practice enhances the process of learning for both students and teacher.

At first glance, it may seem that mathematics and martial arts are conceptually far apart. However, this is not the case. The first thing to understand is that both disciplines are difficult, yet creative, human activities. Martial arts are more than just kicks, punches, and throws; mathematics is not merely a collection of rules, facts, skills, and algorithms. When performed by a skilled practitioner, both are art forms that teach us ways of learning and a framework of thinking that better enables us to use our bodies and minds by maximizing their efficiency. One cannot achieve a high level of skill in mathematics or martial arts by following or executing a collection of rules, facts, and techniques. On the contrary, they are *arts* of exploration, discovery, imagination, and creation. The practitioner enjoys the excitement of searching for new results and techniques, the thrill of discovery, the satisfaction of mastering difficulties, and the pride of achieving mastery.

Mathematics and Martial Arts

It may come as a surprise that learning martial arts requires as much use of the brain as the body. The word *dojo* means the place of enlightenment. The dojo is a place for facing one's weaknesses and for cultivating a flexible mind and body through hard practice. Both martial arts and mathematics are intersections of art, practical skills, and high ideals that provide a structure to develop an awareness of life through a process of discovery (Devlin, 2000; Funakoshi, 1954; Halmos, 1985; Stewart; 2006). Mathematics and martial arts have a fundamental commonality; in order to master either one, the practitioner must become skilled at both the mechanical side and the creative, humanistic side. It is possible to perform both mathematics and martial arts using strict rules of deductions and a system of axioms or techniques where all the theorems or moves are then obtained and checked mechanically. However, if you watch a martial arts competition, you will witness a messy struggle punctuated only occasionally with something as beautiful as the *kata*—a synchronized sequence of combative defensive and offensive techniques in a continuous flow. Achieving the beauty and flow of the kata takes more than simply following a series of pre-determined steps. This same idea to applies to mathematics. The mathematician at work makes "vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof" (Halmos, 1968, p.380).

While solidly built on ancient traditions, countless practitioners have further developed both disciplines by devising and polishing techniques, concepts, and ideas. With every generation, martial arts and mathematics evolve through accumulated knowledge, techniques, concepts, perceptions, and experiences built upon by past practitioners. The concepts and techniques continue to change over time. Not only are new concepts and techniques developed, but at the same time old concepts and techniques are reworked, modified, and redefined (Bolelli, 2008; Davis & Hersh, 1983; Halmos, 1968). The practitioner, therefore, can only gain a proper understanding of martial arts or mathematics through constant practice or study that is not limited to a technical perspective, but also includes a historical and cultural perspective. Learning mathematics and martial arts will have a profound effect on the student since the community plays a powerful role in shaping both the works and lives of

Serkan Hekimoglu has graduate degrees in several disciplines. He teaches mathematics, statistics, and economics courses at a community college. He has practiced wrestling, taekwondo, and karate. He holds a first degree black belt in Shotakan karate.

their practitioners (Bolelli, 2008; Ernest, 1998; Gonzalez, 1989).

One can perceive martial arts and mathematics as an amazing range of mountains without a single peak. We might then describe practicing these arts as climbing an endless mountain, with different routes to the same elevation. As the climber scales the mountain, the view below changes. Able to see more of the surroundings, the climber's sense of where he is and what really exists in the world changes. Continuing to practice martial arts or mathematics, he will find himself able to move his mind in new ways and gradually discover new strengths that can expand his mental horizons. A mountain climber can always try to reach higher elevations or can choose to be content with reaching a certain plateau, even though there are higher peaks in the range. As one climbs higher, the view and the connections between points become more interesting and more intriguing. Martial arts and mathematics also offer many challenges, both external and internal. The difficulty of certain movements, the complexity of the concepts, exhaustion resulting from rigorous practice and study, and the pain of sore muscles or headache can produce a great deal of frustration and discouragement. The journey for each individual is unique. A master or a teacher can illuminate principles behind techniques and concepts but one must discover the truth for oneself.

Mathematics and martial arts are pilgrimages of self-improvement; driven by human desires to find perfection and purity in the human mind and body by uncovering the hidden simplicity and complexity that coexist in the world (Halmos, 1985; Konzak & Bourdeau, 1984). In both disciplines, the knowledge is not so much something that one possesses, but rather is a process of self-discovery. One constructs mathematical ideas or martial arts techniques internally, as a way of dealing with a perceived problem. Therefore, the nature of the objective governs the selection and the use of tools, whether they are legs, arms, concepts, algorithms, or techniques. On the journey toward mastery in mathematics or martial arts, the practitioner learns to combine ideas or techniques through experience, hard work and recognition of what is important. Eventually, the practitioner may feel as though he is no longer simply using tools and concepts as presented to him, instead using their combinations to create something new. Depending on the amount of commitment and energy the practitioner has put into training and studying, there are feelings of hard-won sense of accomplishment, satisfaction, and self-improvement.

Misguided Public Perception of Both Disciplines

Martial arts and mathematics both suffer from a misunderstanding by the general public. Both disciplines have received an unfortunate public image, which is quite different from the perspective of the practitioner (Funakoshi, 1954; Stewart, 2006). While the general public usually considers having a PhD in mathematics or a black belt in martial arts a mark of expertise, the practitioners of both perceive these achievements simply as mere demonstrations of a committed student (Halmos, 1985; Layton, 1988; Stewart, 2006). Furthermore, in addition to being confused regarding the goals of martial arts practice or learning mathematics, the general public is equally clueless as to the true benefits of martial arts and mathematics. Many people view mathematics as an abstract, non-creative, body of knowledge that is to be memorized and applied in a mechanical way (Devlin, 2000; Schoenfeld, 1989). On the contrary, mathematics is a science of patterns which demands creativity. Mathematics requires the use of a vivid imagination, a sense of scientific beauty, and the ability to reason in selecting ideas and concepts (Halmos, 1968). In a similar vein, the true benefit of martial arts does not lie in its sporting value or as a means of fighting, but in the opportunity it provides for becoming a stronger, more complete individual.

Current movies provide a much-distorted picture of what mathematics and martial arts really are, as the philosophy and the subtle beauty of the arts do not come across well on the screen. Movies about mathematics (e.g., *Pi* and *Proof)* frequently provide a negative image of mathematicians by portraying them as loner sociopathic savants. At best, movies may depict a mathematician as an absent-minded nerd engrossed in scribbles and equations, or as a kind of human calculator who can perform complicated mental calculations with amazing speed and accuracy (Burton, 1989; Furinghetti, 1993; Hekimoglu & Kittrell, 2010; Lim, 1999; Mendick, 2002; Picker & Berry, 2000). The negative impact of these movies is their unrealistic representation of the mathematics problem-solving process. For instance, the crime drama *Numb3rs* depicts the main character solving problems in less than a day. However, in reality, a cadre of mathematicians might take months to solve such problems.

In striking comparison, the violent martial arts movies contribute to the corruption of the discipline by portraying the stereotypical image of a martial artist as a bare-handed, acrobatic, *Marlboro Man* who screams with flying exotic high kicks (Layton, 1988; Reiter, 1975). The spectacular and flashy movements that require excellent athletic abilities are highly unrealistic with regard to fighting. Many movies provide a romantic illusion of fighting, along with a fantasy of what it takes to master in martial arts. The portrayal of Daniel Larusso in *The Karate Kid* provides an example of a martial student who trains with a "master" for a short period of time and rapidly becomes proficient in karate-dō (Weintraub & Avildsen, 1984). Even worse, some recent reality shows present an unattractive image of martial artists by portraying them as mindless jocks or buffoons, e.g., *The Ultimate Fighter Reality Show*. Both learning mathematics and training in martial arts are vastly complex endeavors that require intense concentration in order to succeed (Barnfield, 2003; Brown, 2003; Halmos, 1968; Hardman, 1954). The transition from uninformed enthusiast to committed student is a gradual one because it takes time to develop competence by going through a slow and constant contemplative process of change and improvement. Gradually, with practice, reflection, and experience gained through handling different opponents or solving problems, one begins to understand what mathematics or martial arts are really about.

Learning Mathematics vs. Practicing Martial Arts

Ideas and visions form the basis for the practice of both mathematics and martial arts. The process of learning in these disciplines is a series of realizations or awakenings; the harder one studies, the more fascinating the arts become (Bolelli, 2008; Brown, 2003; Gonzalez, 1989; Halmos, 1985; Stewart 2006). The practitioners need to make a healthy obsession of technical details. It is one thing to understand the techniques and concepts, but it is quite another to know them intuitively. In mathematics and martial arts, practitioners must repeat certain movements, techniques, exercises, and algorithms many times so that they can become part of their natural reflexes or thought processes. Practicing a technique or algorithm repeatedly not only makes one more proficient, it also trains and develops his or her neuromuscular or cognitive system to act, respond, or think in accordance with the technique or concept. The outcome of a successful learning experience is either an *assimilation*, the integration of new understanding into the existing neuromuscular or cognitive structures, or an *accommodation*, a reorganization of the existing neuromuscular or cognitive structures in order to allow one to develop these structures on higher levels of organization (Piaget, 1985; Steffe & Wiegel, 1996; von Glasersfeld, 1995). Through a series of assimilations and accommodations, the connections become more interesting and more nuanced. The student's understanding becomes more refined as he or she begins to relate to more subtle dimensions of techniques and algorithms by examining why they work and what constitutes the elements of their effectiveness.

Progressive skills and knowledge development are keys to long-term progress in both mathematics and martial arts since everything that one learns is merely a preliminary foundation for the next level. Learning in martial arts and mathematics is like building a house. A solid foundation is required so that the structural integrity of the house remains intact. Similarly, practitioners need to take the time to build a solid foundation of basic skills and concepts, and constantly refine and add to this base so that they can expand their knowledge. There are neither concepts in mathematics, nor skills in martial arts, that can exist without a foundation. Therefore, failing to develop a proper understanding of fundamental concepts or skills prevents the student from improving and refining his skill level. When one learns a new martial arts technique or a mathematics concept, he or she must incorporate the elementary principles they already know with the new knowledge in order to broaden its scope and applications. When it is difficult to grasp a new step or concept, a student needs to break it down by isolating the appropriate relationships and properties, and then practice or study them separately through continuous self-reflection (Gonzalez, 1989; Hardman, 1954; Skemp, 1971; VonGlasersfeld, 1995). The learning process starts with the introduction of basic concepts or techniques; the instructor then gradually increases the complexity and difficulty of the material as a student advances. In martial arts, the student starts with learning basic punches, kicks, blocks, and stances. Once comfortable with the basics, the student learns how to put them together in kata and fighting practices. Similarly in mathematics, as the student's knowledge grows, new ideas and concepts are introduced that build upon the previous ones.

To become experts in both disciplines, students must not only acquire facts, but also organize their knowledge to facilitate the application to diverse situations. It is this understanding that makes one a mathematical expert or a formidable fighter and enables him to use the knowledge or techniques creatively, flexibly, and fluently, in different settings or problems. The learning process requires the ability to shift attention from the objects or techniques to the structure of their properties and relationships. Later, the student needs to compose parts in such a way that they form a coherent whole. For example, one cannot look at proofs and katas as if they were arbitrary collections of steps or techniques. There is a need to understand each step or technique and how each is related to previous and proceeding ones in the proof or kata. The student should be able to see the proof or kata as a single object by putting the steps back together into one complete object or technique.

Proofs and Katas

Martial arts and mathematics instructors know well the problems that students have appreciating the need to practice katas or complete proofs. We often get frustrated when we hear students saying that practicing katas is boring or that practicing katas does not help them learn how to defend themselves. Neither do we like to hear students questioning the importance of proving mathematical theorems. What functions do katas and proofs have within martial arts and mathematics and what makes the practice of them a meaningful activity? First, katas and proofs provide the glue that holds martial arts and mathematics together; they serve as a means of systematization in both disciplines. In mathematics, proofs help us to systematize various known mathematical results into a deductive system of axioms, definitions, and theorems. In martial arts, a kata unifies techniques by integrating unrelated kicks, punches, and blocks, leading to an aesthetic and efficient presentation of movements. Another function of proofs and katas are that they are forms of discourse. Both serve as a medium for communication and validation of traditions among people who share similar backgrounds (Bolelli, 2008; Hopkins, 2004; Davis, 1976; Funakoshi, 1954; Gale, 1990; Gonobolin, 1954; Hanna, 1989; Tall, 1989).

Katas and proofs also serve as the standard measure of the technical basis of competence. A student’s understanding of martial arts or mathematics can be seen in his performance of the kata or in providing proof of a concept. Additionally, proofs and katas can serve as a challenge. Mathematicians find the process of doing mathematical proofs appealing because they test their knowledge and creativity. To martial artists, katas provide a physical challenge that they find as appealing as the mental challenge of a mathematical proof (Campell, 2005; Manin, 1981; Renz, 1981). Lastly, proofs and katas are teaching and learning tools. Both help to acculturate students in the discipline since they embody lessons learned by past masters (Campell, 2005; Hopkins, 2004; Wilder, 1994). The execution of katas or proofs will provide a student with some of the most effective fighting or mathematical techniques ever developed. The techniques in katas or ideas in proofs can also be a springboard for further techniques or concepts not found in the particular kata or proof under study. They serve as important tools for clarification, validation, and deeper understanding (Bolelli, 2008; Campell, 2005; Fischbein, 1982; Funakoshi, 1954; Gonobolin, 1954; Hopkins, 2004; Tall, 1989; Van Asch, 1993; Van Dormolen, 1977; Volmink, 1990).

Achieving Mastery

Mastery in math and martial arts does not just happen, one achieves mastery over time. Achieving mastery is a slow, gradual, and often frustrating process (Brown, 2003; Hobart, 2006; Stewart, 2006). Thus, patience is an essential quality of both martial artists and mathematicians. Discipline is crucial since the improvement is a gradual, day-by-day process. One can only achieve genuine success by making full use of those valuable experiences sometimes referred to as failures. There is no shame in being knocked down by an opponent or being unable to solve a problem. Once you have learned how to turn pain and frustration into self-knowledge and personal growth, the challenges focus more on what is being learned and how it can be developed more fully. Only those interested in the higher ideal will find martial arts interesting enough to persevere through the rigors it entails (Halmos, 1985; Bolelli, 2008). Those who do will find that the harder they train the more fascinating the art becomes. While martial artists pay for their expertise with sweat, bruises, and blood, mathematicians pay the price with many sleepless nights and headaches. The more time one spends doing mathematics or the harder he trains in martial arts, the more one begins to appreciate the true depth and beauty of each discipline. This new appreciation does not mean that his previous understanding was wrong; it simply means that he has moved on to a higher level (Hardman, 1954; Richman & Rehberg, 1986).

Struggles are also a normal part of both mathematics and martial arts training processes. Without perseverance, there is little chance of ever pushing through the hard times. Breakthroughs result from sustained effort. In both disciplines, the way to true understanding must lead through personal experience and suffering. Even though there are natural stages in the development of a martial artist or a mathematician, it takes effort to move from one to the next. Only those who constantly renew their commitment to study and train with interest and enthusiasm will attain the highest level of achievement. When you hit a wall in your learning, the key to overcoming the barrier is to immerse yourself completely in the problem or technique. As grandmaster Gichin Funakoshi (1954) expressed, “you must be deadly serious in training…I do not mean that you should be reasonably diligent or moderately in earnest” (p. 105). Paul Halmos (1985), one of the leading mathematicians of the twentieth century expressed similar ideas by saying that learning mathematics requires complete focus and loyalty: “To be a mathematician, you must love mathematics more than anything else, more than family, more than religion, more than any other interest” (p. 400).

Overcoming Disillusionment and Attrition

The students of typical martial arts dojos or mathematics classrooms are extremely heterogeneous. Each student brings a unique set of strengths, weaknesses, interests, ambitions, responsibilities, levels of motivation, and approaches to studying or training. Differing physical capacities or mathematical knowledge, emotional maturity, and psychological factors create varying dynamics for each student. This means that instructors in both disciplines must become comfortable with the idea of individualizing instruction for their students. Teachers should adjust the vigor and degree of difficulty in sparring and the difficulty level of mathematics problems to the student’s current developmental level (Piaget, 1985; Vygotsky, 1978). The instructor should adjust and enrich the curricula through differentiations in pace and depth, as well as making changes in their teaching style to match the way students learn. In both disciplines, the instructor’s judgment is extremely important in knowing when to press onward with intensive training to stimulate learning and when to stop in order to avoid student injury or discouragement.

Not surprisingly, attrition remains a significant problem in both endeavors. The slow process of growth is often unbearable to many students who have come to expect instant gratification. Students often have unrealistic expectations of what they can achieve with martial arts or mathematics and how quickly they will be *finished*. Many students want to get their black belts or get an A in their mathematics class to gain a sense of self-confidence and success (Middleton, 1995; Layton, 1988; Reid, Wood, Smith, & Petozc, 2005). Students should practice martial arts and mathematics for their own sake. One must be willing to spend time outside of regular practice or class time to fully internalize the techniques, concepts, algorithms, or movements. In martial arts, the realization that mastery can be achieved from endless training has given way to the more popular fantasy of an easily won black belt status after a few months’ work (Richman & Rehberg, 1986). Likewise, in undergraduate mathematics classes, students generally receive a rude awakening after the first exam when they realize that they cannot begin studying one or two nights before the test and expect to do well on the examinations. Frequently, students will become disillusioned with the amount of hard work required to excel, and so a large percentage of students of both disciplines will drop out or fail (Brown, 2003; Grady, 2000; Hobart, 2006; Jackson & Leffingwell, 1999).

The truth is that there are no shortcuts or magic formulas for learning mathematics or martial arts. The key to success in both disciplines is simply to become personally accountable for what you learn or do not learn, and to practice or study as often and as hard as you can. The skills that look so easy when performed by a master martial artist or a mathematician are not the result of the martial artist’s unique body or the mathematician’s unique mind; their performances are the result of long, hard, and dedicated practices.

Real-World Applications

One should learn the real-world applications of the techniques of martial arts and the concepts of mathematics in context. A single movement or concept will have several different applications, and the ideas and techniques can be adapted to achieve various goals. Bridging the gap between practice and real-world applications will help students to develop a proper understanding of what martial arts or mathematics is and how it relates to the rest of the world. The dilemma is the trade-off between content and real-world problems in mathematics classes or forms and fighting in martial arts classes. To learn the fighting lessons of martial arts, a student must experience a physical encounter through an unchoreographed exchange of techniques (Alsina, 2001; Grady, 2000; Kloosterman, 1996; Olson, 2003; Stewart, 2006). Similarly, students need to see the application of mathemathics in different academic disciplines, where extraneous variables complicate problems or standard algorithms are insufficient.

Teachers must inject realism into a student’s training, because actual violence differs greatly from choreographed training in the dojo, and real problem solving processes differ significantly from the polished proofs in mathematical journals. In both martial arts and mathematics training, it is the instructor's job to challenge students to seek new levels of excellence. In martial arts, the instructor should help students to avoid developing false confidence while working with smaller training partners. Until a student is certain techniques work against a larger person, the student has not learned self-defense. In mathematics classes, students sometimes shy away from working with complex problems and the instructor must challenge students to reach beyond their comfort level and increase their knowledge base. Without knowing the applications of the art, studying mathematics becomes merely a mental exercise, and training in martial arts is no more than exercise for the body.

Teaching Mathematics Based on Martial Arts Principles: A Personal Story

As a longtime martial arts practitioner, I have discovered that I can apply the principles of martial arts to the teaching of mathematics. When free sparring, the goal is to learn from an opponent and to remain deeply attentive. It is imperative to make no assumptions regarding one's own actions or those of the opponent. One must try to develop the correct understanding of the opponent's movements and the proper techniques for responding to them. Translating this basic principle to my life as a mathematics instructor, I strive to be fully present and connected in the classroom. As with martial arts, each teaching moment requires constantly adjusting to the needs of the student. At the beginning of my teaching years, my focus was on the mathematics, not the students. I used to think the students were in the classroom for the mathematics, not that the mathematics was there for the students. If I truly want to motivate my students, then I must find a way to reach their interests. It took me a while to realize that my students do not really care how much mathematics I know. Instead, what they need to see is how much I care about teaching them mathematics. A good instructor must act in harmony with the students, and remember to be the teacher of the students you *actually* have, not the students that you might *wish* to have.

Through my martial arts training, I have learned that it is necessary to develop a sense of self-esteem and mutual respect between the instructor and students in the dojo. A good martial arts instructor never tries to impress students with his own skills and knowledge. His motto is not let me show you what I can do, but rather let's see what we can do together. When I apply this philosophy to the mathematics classes I teach, I know that my mathematical knowledge may give me power, but it is my character that earns the respect of my students. I strive to model excellence for my students. This helps to build trust and respect, and will hopefully encourage them to raise their level of performance. As an instructor, it is essential to be knowledgeable, challenging, organized, clear, and fair. But these characteristics matter little without the desire to encourage students' learning (Jackson & Leffingwell, 1999; Hekimoglu & Kittrell, 2010; Schon, 1987). It is just as important to be committed, enthusiastic, and genuinely warm to motivate students to give me their best, and to encourage them to strive for excellence in everything they do. As my martial arts instructors did for me, my job as a mathematics instructor is to create a stimulating classroom environment that inspires effort and achievement.

Another lesson from my free sparring sessions that I have integrated into my teaching of mathematics is the adoption of basic karate principles of *ikken hissatsu* (finish with one blow). In free sparring, one tries to finish an opponent with one strike without using fancy or complicated maneuvers. In my classes, I create lesson plans based on this principle. I try to pare away anything convoluted and confusing by presenting the concept and ideas in a clear and logically progressive manner. Furthermore, free sparring has taught me to always consider the possibility that I may be unable to conquer my opponent. Likewise, a mathematics instructor should consider the possibility that they may be unable to reach their students with their primary teaching method. When you teach or initiate an attack in free sparring, you should always try to gauge the reaction of your students or opponents before you proceed. The experienced martial artist or mathematics instructor guides his actions by his opponents' or students' reactions.

One of the most important things that I have learned in martial arts training and have integrated into my mathematics teaching is to make my class a place where students can confront their anxieties and fears. To become a good fighter, every martial arts student must learn to face fear. If you attack with the fear of being injured, your attack will not be fully committed and the probability of being injured increases. For students of mathematics, the real enemies are the doubt, confusion, and fear within the students themselves. A student must learn to overcome the frustration, discouragement, and even depression that can result from failure to make satisfactory progress. The presence of fear and anxiety will inhibit the progression of learning (Garofalo, 1989; Hackett & Betz, 1989; Hall & Ponton, 2005; McLeod, 1994).

The essence of teaching mathematics lies in leading students to believe that they can learn mathematics (Crawford, Gordon, Nicholas, and Prosser, 1994; Kloosterman, 1996; McLeod, 1994). They must be able to visualize success, instead of focusing on the chance of failure. I always try to create an open and positive environment where setbacks, mistakes, errors, and failures are permissible. In this way, students can explore their potential without fear of judgment or criticism. I have also learned that testing in both martial arts and mathematics is simply an opportunity to reflect on the student's progress and allow them to acknowledge their strengths, weaknesses, and discover areas for self-improvement. As an instructor, I must help my students to realize their own ability to go beyond their self-imposed limitations.

Training in traditional martial arts is one of the most valuable pursuits I know. The more I began integrating martial arts principles into my teaching, not only did my outward success grow, but more importantly, my sense of being true to myself brought me a deeper satisfaction. Learning to teach mathematics and martial arts training are both ongoing journeys, with each new experience leading to a new challenge. The real secret to becoming an expert in both martial arts and mathematics instruction is realizing that the learning is a process of self-discovery (Bolelli, 2008; Schon, 1987). By striving to perfect one's self-ability and understanding of the abilities of others, wisdom in the discipline develops. The principles of martial arts have become a medium that have given me the means to expand my potential and to enhance the experience of my students.

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