

Guest Editorial...

Why Mathematics? What Mathematics?

Anna Sfard

“Why do I have to learn mathematics? What do I need it for?” When I was a school student, it never occurred to me to ask these questions, nor do I remember hearing it from any of my classmates. “Why do I need history?”—yes. “Why Latin?” (yes, as a high school student I was supposed to study this ancient language)—certainly. But not, “Why mathematics?” The need to deal with numbers, geometric figures, and functions was beyond doubt, and mathematics was unassailable.

Things changed. Today, every other student seems to ask why we need mathematics. Over the years, the quiet certainty of the mathematics learner has disappeared: No longer do young people take it for granted that everybody has to learn math, or at least the particular mathematics curriculum that is practiced with only marginal variations all over the world. The questions, “Why mathematics? Why *so much* of it? Why ‘for all’?,” are now being asked by almost anybody invested, or just interested, in the business of education. Almost, but not all. Whereas the question seems to be bothering students, parents and, more generally, all the “ordinary people” concerned about the current standards of good education, the doubt does not seem to cross the minds of those who should probably be the first to wonder: mathematics educators, policy makers, and researchers. Not only are mathematics educators and researchers convinced about the importance of school mathematics, they also know how to make the case for it. If asked, they will all come up with a

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number of reasons, and their arguments will look more or less the same, whatever the cultural background of its presenter. Yet these common arguments are almost as old as school mathematics itself, and those who use them do not seem to have considered the possibility that, as times change, these arguments might have become unconvincing.

Psychologically, this attitude is fully understandable. After all, at stake is the twig on which mathematics education community has weaved its nest. And yet, as the wonderings about the status of school mathematics are becoming louder and louder, the need for a revision of our reasons can no longer be ignored. In what follows, I respond to this need by taking a critical look at some of the most popular arguments for the currently popular slogan, “Mathematics for all.” This analysis is preceded by a proposal of how to think about mathematics so as to loosen the grip of clichés and to shed off hidden prejudice. It is followed by my own take on the question of what mathematics to teach, to whom, and how.

What Is Mathematics?

To justify the conviction that competence in mathematics is a condition for good citizenship, one must first address the question of what mathematics is and what role it has been playing in the life of the Western society.¹ Here is a proposal: I believe that it might be useful to think about any type of human knowing, mathematics included, as an activity of, or a potential for, telling certain kinds of stories about the world. These stories may sometimes appear far removed from anything we can see or touch, but they nevertheless are believed to remain in close relationship to the tangible reality and, in the final account, are expected to mediate all our actions and improve the ways in which we are going about our human affairs. Since mathematical stories are about objects that cannot be seen, smelled, or touched, it may be a bit difficult to see that the claim of practical usefulness applies to mathematics as much as to physics or biology. But then it suffices to recall the role of, say, measurements and calculations in almost any task a person or a society may wish to undertake to realize that mathematical stories are, indeed, a centerpiece of our universal world-managing toolkit. And I have used just the simplest, most obvious example.

So, as the activity of storytelling, mathematics is not much different from any other subject taught in school. Still, its

narratives are quite unlike those told in history, physics or geography. The nature of the objects these stories are about is but one aspect of the apparent dissimilarity. The way the narratives are constructed and deemed as endorsable (“valid” or “true”) makes a less obvious, but certainly not any less important, difference. It is thus justified to say that *mathematics is a discourse* – a special way of communicating, made unique by its vocabulary, visual means, routine ways of doing things and the resulting set of endorsed narratives – of stories believed to faithfully reflect the real state of affairs. By presenting mathematics in this way (see also Sfard, 2008), I am moving away from the traditional vision of mathematics as given to people by the world itself. Although definitely constrained by external reality, mathematics is to a great extent a matter of human decisions and choices, and of contingency rather than of necessity. This means that mathematical communication can and should be constantly monitored for its effects. In particular, nothing that regards the uses of mathematics is written in stone, and there is no other authority than ourselves to say what needs to be preserved and what must be changed. This conceptualization, therefore, asks for a critical analysis of our common mathematics-related educational practices.

Why Mathematics? Deconstructing Some Common Answers

Three arguments for the status of mathematics as a *sine qua non* of school curricula can usually be heard these days in response to the question of why mathematics: the utilitarian, the political, and the cultural. I will call these three motives “official,” so as to distinguish them from yet another one, which, although not any less powerful than the rest, is never explicitly stated by the proponents of the slogan “mathematics for all.”

The Utilitarian Argument: Mathematics Helps in Dealing With Real-Life Problems

Let me say it again: Mathematics, just as any other domain of human knowledge, is the activity of describing—thus understanding—the world in ways that can mediate and improve our actions. It is often useful to tell ourselves some mathematical stories before we act, and to repeat them as we act, while also forging some new ones. With their exceptionally high level of

abstraction and the unparalleled capacity for generalization, mathematical narratives are believed to be a universal tool, applicable in all domains of our lives. And indeed, mathematics has a long and glorious history of contributions to the well-being of humankind. Ever since its inception, it has been providing us with stories that, in spite of their being concerned with the universe of intangible objects, make us able to deal with the reality around us in particularly effective and useful ways. No wonder, then, that mathematics is considered indispensable for our existence. And yet, whereas this utilitarian argument holds when the term “our existence” is understood as referring to the life of the human society as a whole, it falls apart when it comes to individual lives.

I can point to at least two reasons because of which the utility claim does not work at the individual level. First, it is enough to take a critical look at our own lives to realize that we do not, in fact, need much mathematics in our everyday lives. A university professor recently said in a TV interview that in spite of his sound scientific-mathematical background he could not remember the last time he had used trigonometry, derivatives, or mathematical induction for any purpose. His need for mathematical techniques never goes beyond simple calculation, he said. As it turns out, even those whose profession requires more advanced mathematical competency are likely to say that whatever mathematical tools they are using, the tools have been learned at the job rather than in school.

The second issue I want to point to may be at least a partial explanation for the first: People do not necessarily recognize the applicability of even those mathematical concepts and techniques with which they are fairly familiar. Indeed, research of the last few decades (Brown et al., 1989; Lave, 1988; Lave & Wenger, 1991) brought ample evidence that having mathematical tools does not mean knowing when and how to use them. If we ever have recourse to mathematical discourse, it is usually in contexts that closely resemble those in which we encountered this discourse for the first time. The majority of the school-learned mathematics remains in school for the rest of our lives. These days, all this is known as a manifestation of the phenomenon called *situatedness of learning*, the dependence of the things we know on the context in which they have been learned. To sum up, not only is our

everyday need for school mathematics rather limited, the mathematics we could use does not make it easily fit into our lives. All this pulls the rug from under the feet of those who defend the idea of teaching mathematics to all because of its utility.

The Political Argument: Mathematics Empowers

Because of the universality of mathematics and its special usefulness,² the slogan knowledge is power, which can now be translated into discourses of power, applies to this special form of talk with a particular force. Ever since the advent of modernity, with its high respect for, and utmost confidence in, human reason, mathematics has been one of the hegemonic discourses of Western society. In this positivistically-minded world, whatever is stated in mathematical terms tends to override any other type of argument (just recall, for instance, what counts as decisive “scientific evidence” in the eyes of the politician), and the ability to talk mathematics is thus considered as an important social asset, indeed, a key to success. But the effectiveness of mathematics as a problem-solving tool is only a partial answer to the question of where this omnipotence of mathematical talk comes from. Another relevant feature of mathematics is its ability to impose linear order on anything quantifiable. Number-imbued discourses are perfect settings for decision-making and, as such, they are favored by many, and especially by politicians (and it really does not matter that all too often, politicians can only speak pidgin mathematics; the lack of competency is not an obstacle for those who know their audience and are well aware of the fact that numbers do not have to be used correctly to impress).

The second pro-math argument, one that I called political, can now be stated in just two words: Mathematics empowers. Indeed, if mathematics is the discourse of power, mathematical competency is our armor and mathematical techniques are our social survival skills. When we wonder whether mathematics is worth our effort, at stake is our agency as individuals and our independence as members of society: If we do not want to be pushed around by professional number-jugglers, we must be able to juggle numbers with them and do it equally well, if not better. Add to this the fact that in our society mathematics is a gatekeeper to many coveted jobs and is thus a key to social mobility, and you cannot doubt the universal need for mathematics any longer.

Now it is time for my counter-arguments. The claim that “mathematics empowers” is grounded in the assumption that mathematics is a privileged discourse, a discourse likely to supersede any other, but should the hegemony of mathematics go unquestioned? On a closer look, not each of its uses may be for the good of those whose well-being and empowerment we have in mind when we require “mathematics for all.” For example, when mathematics, so effective in creating useful stories about the physical reality around us, is also applied in crafting stories about children (as in “This is a below average student”) and plays a decisive role in determining the paths their lives are going to take, the results may be less than helpful. More often than not, the numerical tags with which these stories label their young protagonists, rather than empowering the student, may be raising barriers that some of the children will never be able to cross. The same happens when the ability to participate in mathematical discourse is seen as a norm and the lack thereof as pathology and a symptom of a general insufficiency of the child’s “potential.” I will return to all this when presenting the “unofficial” argument for the obligatory school mathematics. For now, the bottom line of what was written so far is simple: we need to remember that by embracing the slogan “mathematics empowers” as is, without any amendments, we may be unwittingly reinforcing social orders we wish to change. As I will be arguing in the concluding part of this editorial, trying to change the game may be much more “empowering” than trying to make everybody join in and play it well.

The Cultural Argument: Mathematics Is a Necessary Ingredient of Your Cultural Makeup

In the last paragraph, I touched upon the issue of the place of mathematics in our culture and in an individual person’s identities. I will now elaborate on this topic while presenting the cultural argument for teaching mathematics to all.

Considering the fact that to think means to participate in some kind of discourse, it is fair to say that our discourses, those discourses in which each of us is able to participate, constitute who we are as social beings. In the society that appreciates intellectual skills and communication, the greater and more diverse our discursive repertoire, the richer, more valued, and

more attractive our identities. However, not all discourses are made equal, so the adjective “valued” must be qualified. Some forms of communicating are considered to be good for our identities and some others much less so. As to mathematics, many would say that it belongs to the former category. Considered as a pinnacle of human intellectual achievement and thus as one of the most precious cultural assets, it bestows some of its glory even on peripheral members of the mathematical community. Those who share this view believe that mathematical competency makes you a better person, if only because of the prestigious membership that it affords. A good illustration of this claim comes from an Israeli study (Sfard & Prusak, 2005) in which 16-year-old immigrant students, originally from the former Soviet Union, unanimously justified their choice of the advanced mathematics program with claims that mathematics is an indispensable ingredient of one’s identity, saying, for example, “Without mathematics, one is not a complete human being.”

But the truth is that the attitude demonstrated by those immigrant students stands today as an exception rather than a rule. In the eyes of today’s young people, at least those who come from cultural backgrounds I am well acquainted with, mathematics does not seem to have the allure it had for my generation. Whereas this statement can be supported with numbers that show a continuous decline in percentages of graduates who choose to study mathematics (or science)—and currently, this seems to be a general trend in the Western world³—I can also present some first-hand evidence. In the same research in which the immigrant students declared their need for mathematical competency as a necessary ingredient of their identities, the Israeli-born participants spoke about mathematics solely as a stepping stone for whatever else they would like to do in the future. Such an approach means that one can dispose with mathematics once it has fulfilled its role as an entrance ticket to preferable places. For the Israeli-born participants, as for many other young people these days, mathematical competency is no longer a highly desired ingredient of one’s identity.

Considering the way the world has been changing in the last few decades it may not be too difficult to account for this drop in the popularity of mathematics. One of the reasons may be the fact that mathematical activity does not match the life experiences

typical of our postmodern communication-driven world. As aptly observed in a recent book by Susan Cain (2012), the hero of our times is a vocal, assertive extrovert with well-developed communicational skills and insatiable appetite for interpersonal contact. Although there is a clear tendency, these days, to teach mathematics in collaborative groups—the type of learning that is very much in tune with this general trend toward the collective and the interpersonal—we need to remember that one cannot turn mathematics into a discourse-for-oneself unless one also practices talking mathematics to oneself. And yet, as long as interpersonal communication is the name of the game and a person with a preference for the intra-personal dialogue risks marginalization, few students may be ready to suspend their intense exchanges with others for the sake of well-focused conversation with themselves.

In spite of all that has been said above, I must confess that the cultural argument is particularly difficult for me to renounce. I have been brought up to love mathematics for what it is. Born into the modernist world ruled by logical positivism, I believed that mathematics must be treated as a queen even when it acts as a servant. Like the immigrant participants of Anna Prusak's study, I have always felt that mathematics is a valuable, indeed indispensable, ingredient of my identity—an element to cherish and be proud of. But this is just a matter of emotions. Rationally, there is little I can say in defense of this stance. I am acutely aware of the fact that times change and that, these days, modernist romanticism is at odds with postmodernist pragmatism. In the end, I must concede that the designation of mathematics as a cultural asset is not any different than that of poetry or art. Thus, however we look at it, the cultural argument alone does not justify the prominent presence of mathematics in school curricula.

The Unofficial Argument: Mathematics Is a Perfect Selection Tool

My last argument harks back to the abuses of mathematics to which I hinted while reflecting on the statement “mathematics empowers.” I call it “unofficial,” because no educational policy maker would admit to its being the principal, if not the only, motive for his or her decisions. I am talking here about the use of school mathematics as a basis for the measuring-and-labeling practices mentioned above. In our society, grades in mathematics

serve as one of the main criteria for selecting school graduates for their future careers. Justifiably or not, mathematics is considered to be the *lingua franca* of our times, the universal language, less sensitive to culture than any other well-defined discourse. No intellectual competency, therefore, seems as well suited as mathematics for the role of a universal yardstick for evaluating and comparing people. Add to this the common conviction that “Good in math = generally brilliant” (with the negation being, illogically, “not good in math = generally suspect”), and you begin realizing that teaching mathematics and then assessing the results may be, above all, an activity of classifying people with “price tags” that, once attached, will have to be displayed whenever a person is trying to get access to one career or another. I do not think that an elaborate argument is needed to deconstruct this kind of motive. The very assertion that this harmful practice is perhaps the only reason for requiring mathematics for all should be enough to make us rethink our policies.

What Mathematics and Why? A Personal View

It is time for me to make a personal statement. Just in case I have been misunderstood, let me make it clear: I do care for mathematics and I am as concerned as anybody about its future and the future of those who are going to need it. All that I said above grew from this very genuine concern. By no means do I advocate discontinuing the practice of teaching mathematics in school. All I am trying to say is that we should approach the task in a more flexible, less authoritarian way, while giving more thought to the question of how much should be required from all and how much choice should be left to the learner. In other words, I propose that we rethink school mathematics and revise it quite radically. As I said before, if there is a doubt about the game being played, let us change this game rather than trying to play it well. These days, deep, far-reaching change is needed in what we teach, to whom, and how.

I do have a concrete proposal with regard to what we can do. But let me precede this discussion with two basic “don’t’s. First, let us not use mathematics as a universal instrument for selection. This practice hurts the student and it spoils the mathematics that is being learned. Second, let us not force the traditional school

curriculum on everybody, and, whatever mathematics we do decide to teach, let us teach it in a different way.

In the rest of this editorial, let me elaborate on this latter issue, which, in more constructive terms, can be stated as follows: Yes, let us teach everybody *some* mathematics, the mathematics whose everyday usefulness is beyond question. Arithmetic? Yes. Some geometry? Definitely. Basic algebra? No doubt. Add to this some rudimentary statistics, the extremely useful topic that is still only rarely taught in schools, and the list of what I consider as “mathematics for all” is complete. And what about trigonometry, calculus, linear algebra? Let us leave these more advanced topics as electives, to be chosen by those who want to study them.

But the proposed syllabus does not, per se, convey the idea of the change I had in mind when claiming the need to rethink school mathematics. The question is not just of what to teach or to whom, but also of how to conceptualize what is being taught so as to make it more convincing and easier to learn. There are two tightly interrelated ways in which mathematics could be framed in school as an object of learning: we can think about mathematics as the *art of communicating* or as one of the basic forms of *literacy*. Clearly, both these framings are predicated on the vision of mathematics as a discourse. Moreover, a combination of the two approaches could be found so that the student can benefit from both. Let me briefly elaborate on each one of the two framings.

Mathematics as the Art of Communicating

As a discourse, mathematics offers special ways of communicating with others and with oneself. When it comes to the effectiveness of communication, mathematics is unrivaled: When at its best, it is ambiguity-proof and has an unparalleled capacity for generalization. To put it differently, mathematical discourse appears to be infallible—any two people who follow its rules must eventually agree, that is, endorse the same narratives; in addition, this discourse has an exceptional power of expression, allowing us to say more with less.

I can see a number of reasons why teaching mathematics as *the art of communicating* may be a good thing to do. First, it will bring to the fore the interpersonal dimension of mathematics: the word *communication* reminds us that mathematics originates in a conversation between mathematically-minded thinkers, concerned

about the quality of their exchange at least as much as about what this exchange is all about. Second, the importance of the communicational habits one develops when motivated by the wish to prevent ambiguity and ensure consensus exceeds the boundaries of mathematics. I am prepared to go so far as to claim that if some of the habits of mathematical communication were regulating all human conversations, from those that take place between married couples to those between politicians, our world would be a happier place to live. Third, presenting mathematics as the art of interpersonal communication is, potentially, a more effective educational strategy than focusing exclusively on intra-personal communication. The interpersonal approach fits with today’s young people’s preferences. It is also easier to implement. After all, shaping the ways students talk to each other is, for obvious reasons, a more straightforward job than trying to mould their thinking directly. Fourth, framing the task of learning mathematics as perfecting one’s ability to communicate with others may be helpful, even if not sufficient, in overcoming the situatedness of mathematical learning. Challenging students to find solutions that would convince the worst skeptic will likely help them develop the life-long habit of paying attention to the way they talk (and thus think!). This kind of attention, being focused on one’s own actions, may bring about discursive habits that are less context-dependent and more universal than those that develop when the learner is almost exclusively preoccupied with mathematical objects. There may be more, but I think these four reasons should suffice to explain why teaching mathematics as an art of communication appears to be a worthy endeavor.

Mathematics as a Basic Literacy

While teaching mathematics as an art of communicating, we stress the question of *how* to talk. Fostering mathematical literacy completes the picture by emphasizing the issues of *when* to talk mathematically and *what about*.

Although, nowadays, mathematical literacy is a buzz phrase, a cursory review of literature suffices to show that there is not much agreement on how it should be used. For the sake of the present conversation, I will define mathematical literacy as the ability to decide not just about *how* to participate in mathematical discourse but also about *when* to do so. It is the emphasis on the word *when*

that signals that mathematical literacy is different from the type of formal mathematical knowledge that is being developed, in practice if not in principle, through the majority of present-day curricula. These curricula offer mathematics as, first and foremost, a self-sustained discourse that speaks about its own unique objects and has little ties to anything external. Thus, they stress the *how* of mathematics to the neglect of the *when*. Mathematical literacy, in contrast, means the ability to engage in mathematical communication whenever this may help in understanding and manipulating the world around us. It thus requires fostering the *how* and the *when* of the mathematical routines at the same time. To put it in discursive terms, along with developing students' participation in mathematical discourse, we need to teach them how to combine this discourse with other ones. Literacy instruction must stress students' ability to switch to the mathematical discourse from any other discourse whenever appropriate and useful, and it has to foster the capacity for incorporating some of the meta-mathematical rules of communication into other discourses.

My proposal, therefore, is to replace the slogan “mathematics for all” with the call for “mathematical literacy for all.” Arithmetic, geometry, elementary algebra, the basics of statistics—these are mathematical discourses that, I believe, should become a part and parcel of every child's literacy kit. This is easier said than done, of course. Because of the inherent situatedness of learning, the call for mathematical literacy presents educators with a major challenge. The question of how to teach for mathematical literacy must be theoretically and empirically studied. Considering the urgency of the issue, such research should be given high priority.

In this editorial, I tried to make the case for a change in the way we think about school mathematics. In spite of the constant talk about reform, the current mathematical curricula are almost the same in their *content* (as opposed to pedagogy) as they were decades, if not centuries, ago. Times change, but our general conception of school mathematics remains invariant. As mathematics educators, we have a strong urge to preserve the kind of mathematics that has been at the center of our lives ever since

our own days as school students. We want to make sure that the new generation can have and enjoy all those things that our own generation has seen as precious and enjoyable. But times do change, and students' needs and preferences change with them. With the advent of knowledge technologies that allow an individual to be an agent of her own learning, our ability to tell the learner what to study changes as well. In this editorial, I proposed that we take a good look at our reasons and then, rather than imposing one rigid model on all, restrict our requirements to a basis from which many valuable variants of mathematical competency may spring in the future.

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The Roles They Play: Prospective Elementary Teachers and a Problem-Solving Task

Valerie Sharon

The transition from learner to teacher of mathematics is often a difficult one for prospective elementary teachers to negotiate. Learning to teach necessitates the opportunity to practice the discourse of teacher of mathematics. The undergraduate mathematics content classroom provides a setting for prospective teachers to practice the discourse of teacher through their interactions with each other while also learning the mathematical concepts presented in class. This qualitative study sought to examine what roles prospective teachers adopt while engaged in a cooperative problem-solving task. Discourse analysis was applied to analyze the verbal interactions between three participants in a mathematics content course. Key disruptions in the conversation revealed instances of the fluid relationship between learner and teacher of mathematics in the roles they adopted while solving an application problem: self as learner-in-teacher, collaborator as learner-in-teacher, and unlikely learner-in-teacher. The presence of this fluid relationship led to the proposal of a model of learner-in-teacher-in-learner of mathematics. This proposed model suggests that prospective teachers have the opportunity to learn how to teach in and through each other when given the opportunity to engage in dialogue with one another.

The shift from learner of mathematics to teacher of mathematics usually begins in the prospective elementary teacher's mathematics content classroom. Up to this point, the prospective elementary teacher has taken part in the mathematics community as a learner of mathematics and now hopes to take on the role as teacher of mathematics. In the mathematics content classroom, the prospective teacher is expecting to learn both mathematical concepts and how to teach them effectively. The individual in this transitory space is "learning about becoming...by participation in practices" (Lerman, 2001, p. 88).

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- ¹ I am talking about the Western society because due to my personal background, this is the only one I feel competent to talk about. The odds are, however, that in our globalized world there is not much difference, in this respect, between the Western society and all the others.
- ² Just to make it clear: the former argument that mathematics is not necessarily useful in every person's life does not contradict the claim about its general usefulness!
- ³ As evidenced by numerous publications on the drop in enrollment to mathematics-related university subjects (e.g. Garfunkel & Young, 1998; Gilbert, 2006; OECD, 2006) and by the frequent calls for research projects that examine ways to reverse this trend (see e.g. TISME initiative in UK, <http://tisme-scienceandmaths.org/>), the decline in young people's interest in mathematics and science is generally considered these days as one of the most serious educational problems, to be studied by educational researchers and dealt with by educators and policy makers.